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A NEW TECHNIQUE FOR SOLVING ASSIGNMENT PROBLEM USING RANKING EXPONENTIAL VALUE OF PENTAGON FUZZY NUMBERS

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ABSTRACT

In the field of research several techniques are focused for solving Assignment Problems in Fuzzification way. Fuzzy numbers has master role to solve real life problem especially Assignment problem (AP), which finds many applications in allocation and scheduling, generally it made on one – to – one basis and if there are more jobs to do than can be done, one can decide which job to leave undone or what resource to add. But in this paper we introduce a new *technique for solving AP using cardinality of exponential value of Fuzzy Pentagon Number, and we compare our result with previous scholars result in the same area.*

Key Words: Exponential Number, Ranking of Pentagon Fuzzy Number, cardinality, Row reduction, Column Reduction

1. Introduction

Assignment problem (AP) is used in worldwide to solve real world problems. An AP plays an important role in industry and other applications. AP is very often used in solving problems of engineering and management studies

Ranking fuzzy numbers are used as an important tool in Decision Making. In Fuzzy decision analysis, fuzzy quantities are used to determine the performance s of alternatives in modeling real world problems. Various ranking procedures have been developed in 1976 , when the theory of fuzzy sets were first introduced by Zadeh [9] Ranking fuzzy numbers were first proposed by Jain for decision making in fuzzy subsets. In this article we introduce a new approach for ranking exponential value of pentagon fuzzy numbers with Cardinality Concepts, using to solve Assignment Problem to get best optimal feasible solution for AP.

2. Previous Research

Firstly the Ranking Fuzzy Sets, Subsets and Systems were introduced by Bortolan G., Degani, R. in 1989 [2]. Then Chen S. H. 2007 [3] discussed about Ranking Fuzzy Numbers with Maximizing Set and Minimizing Set

Amith Kumar, Singh P, Kaur A. in 2010 [1] approached for Ranking of Generalized Trapezoidal Fuzzy numbers to solve Assignment Problem

Salim Rezvani [4] tried Graded Mean Representation Method with Triangular Fuzzy Numbers to solve Assignment Problem in 2010 and then he approached in Multiplication Operation on Trapezoidal Fuzzy Numbers in 2011 [5]. Afterwords in 2012 [6], he introduced a new method for ranking in Perimeters of two generalized Trapezoidal Fuzzy Numbers for solving Assignment problems

Salim Rezvani, M. Molani, and M. Ebrahimi (2013) et.al [7] developed a new Method for Ranking area in two Generalized Trapezoidal Fuzzy Numbers to solve Assignment Problem finally he solved Assignment Problem by Salim Rezvani in 2014 [8] by Ranking Exponential Fuzzy Numbers by Median Value

3. Preliminaries and Basic concepts

3.1. Interval Number

Let $\mathbb{R}$, it is the set of real numbers and then the closed interval $[a, b]$, it is said to be an interval number, where $a, b \in \mathbb{R}, with\ a \leq b$

3.2. Normal Fuzzy Set

A fuzzy set A , it is the universe of discourse X , and it is called Normal fuzzy set implying that there exists at least one $x \in X$, such that $\mu_A(x) = 1, \mu$, it is any fuzzy number in $[0, 1]$

3.3. Distance between Two Interval Numbers

Let $a = [a_1, a_2], b = [b_1, b_2]$, they are two interval numbers, and then the distance between $a\ and\ b$, denoted by $d(a, b)$, and defined by

$$d(a, b) = \int_{-0.5}^{0} \left\{ \left[\frac{(a_1 + a_2)}{2} + x(a_2 - a_1) \right] - \left[\frac{(b_1 + b_2)}{2} + x(b_2 - b_1) \right] \right\} dx$$

3.4. Properties of fuzzy number and Fuzzy set (without proof)

To qualify as a fuzzy number α and a fuzzy set $A\ on\ \mathbb{R}$, must possess at least the following three properties

a. A, it must be a fuzzy set
b. α_A, it must be a closed interval and for every $\alpha \in [0, 1]$
c. The support of A, denoted by O_A^+, it must be bounded

3.5.The Generalized Fuzzy Number Conditions:

Generally, a general fuzzy number A, it is described as any fuzzy subset of the real line $\mathbb{R}$, whose membership function μ_A satisfies the following conditions:

a. μ_A, it is a continuous mapping from $\mathbb{R}$ to the closed interval $[0, 1]$
b. $\mu_A(x) = 0\ for -\infty \leq x \leq a$
c. $\mu_A(x) = L_1(x)$, it is strictly increasing function on $[a, b]$
d. $\mu_A(x) = L_2(x)$, it is strictly increasing function on $[b, c]$
e. $\mu_A(x) = W\ if\ x = c$
f. $\mu_A(x) = R_1(x)$, it is strictly decreasing function on $[c, d]$
g. $\mu_A(x) = R_2(x)$, it is strictly decreasing function on $[e, \infty]$

Where $0 \leq W \leq 1; a, b \in \mathbb{R}; c, d \in \mathbb{R}^+$

3.6.Generalized Exponential Fuzzy Number

We denote the generalized exponential fuzzy number (say)

$B = (a, b, c, d, e;\ w)_{Exp}\ when\ w = 1$

3.7.Integral Value of Graded Mean $h - level$

Based on the integral value of graded mean $h - level$, however, and this Exponential fuzzy numbers denoted by$f_E(x)$ always have a fix range as $[c, d]$, and we define their general forms as follows:

$$f_E(x) = f(x) = \begin{cases} wexp^{-\left[\frac{(b-x)}{(b-a)}\right]},\ for\ a \leq x \leq b \\ wexp^{-\left[\frac{(c-x)}{(d-c)}\right]},\ for\ b \leq x \leq c \\ w,\ x = c \\ wexp^{-\left[\frac{(x-c)}{(d-c)}\right]},\ for\ c \leq x \leq d \\ wexp^{-\left[\frac{(x-d)}{(e-d)}\right]},\ for\ x > e \end{cases}$$

Where $0 \leq W \leq 1; a, b \in \mathbb{R}; c, d \in \mathbb{R}^+$

From this exponential fuzzy numbers, let us take monotonic functions are

$L_1(x) = wexp^{-\left[\frac{(b-x)}{(b-a)}\right]}$

$L_2(x) = wexp^{-\left[\frac{(c-x)}{(d-c)}\right]}$

$R_1(x) = wexp^{-\left[\frac{(x-c)}{(d-c)}\right]}$

$R_2(x) = wexp^{-\left[\frac{(x-d)}{(e-d)}\right]}$

3.8.Ranking of Pentagon Fuzzy Number

Let us take two pentagon fuzzy numbers denoted by

$\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)\ and\ \tilde{B}_P = (b_1, b_2, b_3, b_4, b_5)$, and then we have

$\tilde{A}_P = \tilde{B}_P \Leftrightarrow R(\tilde{A}_P) = R(\tilde{B}_P)$

$\tilde{A}_P \geq \tilde{B}_P \Leftrightarrow R(\tilde{A}_P) \geq R(\tilde{B}_P)$

$\tilde{A}_P \leq \tilde{B}_P \Leftrightarrow R(\tilde{A}_P) \leq R(\tilde{B}_P)$

Graphical Representation of Pentagon Fuzzy Number

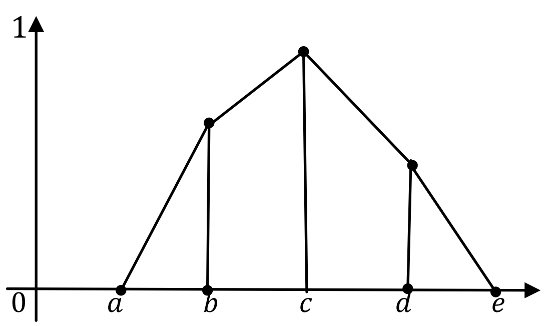

Figure – 3: Graphical Representation of Pentagon Number

Figure – 3: Graphical Representation of Pentagon Number

Now we want to prove two theorems for Cardinality of an exponential Pentagon fuzzy number

3.9.Theorem – 1 for Cardinality of an exponential Pentagon fuzzy number

Cardinality of any exponential pentagon fuzzy number$\tilde{B}_P = (a, b, c, d, e)$, it is characterized by the value of the integral that

$Card\ (B) = \frac{w}{e^1\{[(b-a)(e^1-1)]+[(c-b)](e^1-1)+[(c-d)(e^1-1)]\}} + w$

Proof:

Since $Card\ (B) = \int_a^\infty B(x)dx$

Since $\int_a^\infty B(x)dx = \int_a^b B(x)dx + \int_b^c B(x)dx + \int_c^d B(x)dx + \int_e^\infty B(x)dx + w$

$\Rightarrow \int_a^\infty B(x)dx = \int_a^b we^{-\frac{(b-x)}{(b-a)}} dx + \int_b^c we^{-\frac{(c-x)}{(c-b)}}dx + \int_c^d we^{-\frac{(x-c)}{(e-d)}}dx + \int_e^\infty we^{-\frac{(x-d)}{(e-d)}}dx$

$\Rightarrow \int_a^\infty B(x)dx = \begin{cases} w\left[(b-a)\left(1-\frac{1}{e^1}\right)\right] + w\left[(c-b)\left(1-\frac{1}{e^1}\right)\right] + \\ w\left[(d-c)\left(1-\frac{1}{e^1}\right)\right] + w\left[(e-d)\left(1-\frac{1}{e^1}\right)\right] + w \end{cases}$

$\Rightarrow \int_a^\infty B(x)dx = \frac{w}{e^1\{[(b-a)(e^1-1)]+[(c-b)](e^1-1)+[(c-d)(e^1-1)]\}} + w$

$\Rightarrow\ Card\ (B) = \frac{w}{e^1\{[(b-a)(e^1-1)]+[(c-b)](e^1-1)+[(c-d)(e^1-1)]\}} + w$

Hence complete the proof

3.10. Theorem – 2 for Cardinality of an exponential Pentagon fuzzy number

If B , it is a exponential pentagon fuzzy number with light tail then

$Card\ (B) = we + \frac{w}{3e^1\left[(d-e)(e^1-1)-[(b-a)(1-e^1)]\right]}$

Proof:

Since $Card\ (B) = \frac{w}{e^1\{[(b-a)(e^1-1)]+[(c-b)](e^1-1)+[(c-d)(e^1-1)]\}} + w$ (By theorem – 1)

Since the Exponential fuzzy number with light tail we have

$Card\ (B) = we + \frac{1}{3}\left[\int_e^\infty B(x)dx - \int_a^b B(x)dx\right]$

$$\Longrightarrow Card\,(B) = we + \frac{1}{3}\left[\int_e^{\infty} we^{-\frac{(x-d)}{(c-d)}}dx - \int_a^b we^{-\frac{(b-x)}{(b-a)}}dx\right]$$

$$\Longrightarrow Card\,(B) = we + \frac{1}{3}\left\{w\left[(e-d)\left(1-\frac{1}{e^1}\right)\right] - w\left[(b-a)\left(1-\frac{1}{e^1}\right)\right]\right\}$$

$$\Longrightarrow Card\,(B) = we + \frac{w}{3e^1[(d-e)(e^1-1)-[(b-a)(1-e^1)]]}$$

Hence complete the proof

3.11. Proposed Approach

In this section we explain some important results by using one Numerical Example of Fuzzy Assignment Problem solving by using the Concepts of Cardinality of Exponential Pentagon Fuzzy Number we will get Very Useful Important Result that is mostly useful for the proposed approach is proved

3.12. Numerical Example for Proposed Approach

Solve the following assignment problem of minimal cost by using above proposed approach method

$B =$

$$\begin{bmatrix} (0.2, 0.5, 0.3, 0.4, 0.1; 0.35) & (0.1, 0.3, 0.4, 0.5, 0.6; 0.2) & (0.2, 0.3, 0.5, 0.4, 0.1; 0.1) & (0.1, 0.2, 0.3, 0.5, 0.6; 0.4) \\ (0.1, 0.2, 0.3, 0.5, 0.6; 0.21) & (0.2, 0.4, 0.6, 0.8, 0.9; 0.5) & (0.3, 0.4, 0.5, 0.6, 0.7; 0.28) & (0.1, 0.15, 0.2, 0.25, 0.6; 0.48) \\ (0.1, 0.2, 0.3, 0.5, 0.6; 0.4) & (0.2, 0.3, 0.5, 0.4, 0.1; 0.1) & (0.2, 0.4, 0.7, 0.75, 0.8; 0.39) & (0.1, 0.15, 0.2, 0.25, 0.6; 0.48) \\ (0.1, 0.2, 0.3, 0.4, 0.6, ; 0.51) & (0.1, 0.2, 0.4, 0.1, 0.7; 0.5) & (0.1, 0.2, 0.4, 0.6, 0.7; 0.3) & (0.1, 0.2, 0.3, 0.4, 0.6, 0.25) \end{bmatrix}$$

Solution:

Since by theorem – 2, we have $Card\,(B) = we + \frac{w}{3e^1[(d-e)(e^1-1)-[(b-a)(1-e^1)]]}$

And also we have $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix}$

For example:

Let $b_{11} = (0.2, 0.5, 0.3, 0.4, 0.1; 0.35)$, in this case we have

$a = 0.2, b = 0.5, c = 0.3, d = 0.4, e = 0.1$ and $w = 0.35$ $with$ $e^1 \approx 2.72$ (Approximately)

$\Longrightarrow\ Card\ (b_{11}) = (0.35 \times 0.3) + \frac{0.35}{(3\times2.72)[(0.4-0.1)(2.72-1)-(0.5-0.2)(1-2.72)]}$

$\Longrightarrow\ Card\ (b_{11}) = 0.06$

Similarly we can calculate

$Card\ (b_{12}) = 0.07;\ Card\ (b_{13}) = 0.04\ and\ Card\ (b_{14}) = 0.1$

$Card\ (b_{21}) = 0.07;\ Card\ (b_{22}) = 0.27, Card\ (b_{23}) = 0.12\ and\ Card\ (b_{24}) = 0.05$

$Card\ (b_{31}) = 0.12;\ Card\ (b_{32}) = 0.04, Card\ (b_{33}) = 0.26\ and\ Card\ (b_{34}) = 0.05$

$Card\ (b_{41}) = 0.03;\ Card\ (b_{42}) = 0.2, Card\ (b_{43}) = 0.16\ and\ Card\ (b_{44}) = 0.15$

$$\Longrightarrow\ Card\ (B) = Card\left(\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix}\right)$$

$$\Longrightarrow\ Card\ (B) = \begin{bmatrix} 0.06 & 0.07 & 0.04 & 0.1 \\ 0.07 & 0.27 & 0.12 & 0.05 \\ 0.12 & 0.04 & 0.26 & 0.05 \\ 0.03 & 0.2 & 0.16 & 0.15 \end{bmatrix}$$

Using Row Reduction and Column Reduction Operations by the following rules

a. In the effectiveness matrix, subtract the minimum element of each row from all the elements of the row.
b. See if there is at least one zero in each row and in each column. If it is so, stop here that is
c. Subtract the minimum element of the column not containing a zero element from all the elements of the column

Following these above three rules we get the results, that is,

In the first row the minimum element is 0.04

In the second row the minimum element is 0.05

In the third row the minimum element is 0.04

In the fourth row the minimum element is 0.03 and then subtract each minimum element in each row with other elements in the respective rows we get

$$Card\ (B) = \begin{bmatrix} 0.06-0.04 & 0.07-0.04 & 0.04-0.04 & 0.1-0.4 \\ 0.07-0.05 & 0.27-0.05 & 0.12-0.05 & 0.05-0.05 \\ 0.12-0.04 & 0.04-0.04 & 0.26-0.04 & 0.05-0.04 \\ 0.03-0.03 & 0.2-0.03 & 0.16-0.03 & 0.15-0.03 \end{bmatrix}$$

By Row Reduction

$$Card\ (B) = \begin{bmatrix} 0.02 & 0.03 & [\mathbf{0.00}] & 0.06 \\ 0.02 & 0.22 & 0.07 & [\mathbf{0.00}] \\ 0.08 & [\mathbf{0.00}] & 0.22 & 0.01 \\ [\mathbf{0.00}] & 0.17 & 0.13 & 0.12 \end{bmatrix}$$

The resulting row reduced matrix have at least one zero element in each row

Check if there is at least one zero in each column that is

By Column Reduction

$$Card\ (B) = \begin{bmatrix} 0.02 & 0.03 & [\mathbf{0.00}] & 0.06 \\ 0.02 & 0.22 & 0.07 & [\mathbf{0.00}] \\ 0.08 & [\mathbf{0.00}] & 0.22 & 0.01 \\ [\mathbf{0.00}] & 0.17 & 0.13 & 0.12 \end{bmatrix}$$

Therefore stop the iteration here, because row reduction and column reduction matrices in each row and in each column they have only one zero element.

Examine the Row Reduction or Column Reductions for single $[\mathbf{0.00}]$ and any other zeros in their column or row respectively.

Now the rules of Assignment problem there may no row and no column without assignment that is there is one assignment can be made in the current solution that is the current feasible solution is an optimal solution

Now we have Optimal Minimal Cost is

$b_{13} \times 1 + b_{24} \times 1 + b_{32} \times 1 + b_{42} \times 1 = 0.04 + 0.05 + 0.04 + 0.03 = 0.16$ Units

That is $Card\ (B) = 0.16$ Units

Table for Comparison of Assignment Problem Solving through Ranking with Different Approach for Same Numerical Example

Approaches	**Researchers**	**Results for Ranking using Assignment Problem**
Ranking of Generalized Trapezoidal Fuzzy Numbers	AmithKumar, P.Singh A. Kaur (2010) et. al	$R(B) = 0.23$
A new method for Ranking in Perimeters of two generalized Trapezoidal Fuzzy Numbers	Salim Rezvani (2012) et.al	$R(B) = 0.27$

A new Method for Ranking area in two Generalized Trapezoidal Fuzzy Numbers	Salim Rezvani, M. Molani, and M. Ebrahimi (2013) et.al.	$R(B) = 0.2$
Ranking Exponential Fuzzy Numbers by Median Value	Salim Rezvani (2014) et.al	$R(B) = 1$
Proposed Approach Ranking Pentagon Exponential Fuzzy Numbers By Cardinality concepts Approach	Prof. A. N. Mohamad, Gebreheiwot and Olana Abu (2018)	$\boldsymbol{R(B) = 0.16}$

4. Result Discussion and Conclusion

In this article we choose assignment problem with fuzzy cost coefficient, for the uncertainties factors in real life situations are presented as fuzzy pentagon numbers and we evaluate cardinality concept. We convert the value ranking pentagon exponential fuzzy numbers by cardianlity fuzzy pentagon numbers for fuzzy cost coefficients by using row reduction and column reduction algorithm to solve assignment problem. We also compare our result with other scholars what they done in solving fuzzy assignment problem. If any scholars need to improve their result to get improve solution by using other ranking functions with different ideas. Method other than row reduction and column reduction can also be used to solve assignment problem, but using Fuzzification numbers or fuzzy variables might be more effective to find better solution in fuzzy form.

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