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βS-REGULAR SPACES AND βS-NORMAL SPACES IN TOPOLOGY

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ABSTRACT

The aim of this paper is to introduce and study some forms of weak regular spaces and weak forms of normal spaces , viz. βs- regular spaces , sβ-regular spaces , βs-normal spaces by using β-closed sets and semiopen sets . Also , we studied some related functions like βgs-closed functions , βgs-continuous functions for preserving these regular spaces and normal spaces .

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βgs-closedness , βs-continuity, β-continuity, β-irresoluteness, irresoluteness

1.Introduction

In 1963, N.Levine [16] introduced and studied the concepts of semiopen sets and semi continuity in topological spaces and in 1983, M.E. Abd El–Monsef et al [1] have introduced the concepts of β-open sets and β-continuity in topology . Latter , these β-open sets are recalled as semipreopen sets , which were introduced by D. Andrejevic in [5]. Further these semi open sets and β-open sets (= semipreopen sets) have been studied by various authors in the literature see [2, 4,10,14,17,23]. In 1970 and 1990, respectively Levine [15] and S.P.Arya et al [6] have defined and studied the concept of g-closed sets and gs-closed sets in topology . The aim of this paper is to introduce and study some forms of weak regular spaces and weak forms of normal spaces , viz. βs- regular spaces , sβ-regular spaces , βs-normal spaces by using β-closed sets and semiopen sets . Also , we studied some related functions like βgs-closed functions , βgs-continuous functions for preserving these regular spaces and normal spaces .

2. Preliminaries

Throughout this paper X,Y will denote topological spaces on which no separation axioms assumed unless explicitly stated. Let $f : X \to Y$ represent a single valued function. Let A be a subset of X. The closure and interior of A are respectively denoted by Cl(A) and Int(A).

The following definitions and results are useful in the sequel.

Definition 2.1: A subset A of X is called

(i) semiopen (in short , s-open) set [16] if $A \subset ClInt(A)$.

(ii) preopen (in short , p-open) set [20] if $A \subset IntCl(A)$.

(iii) β-open [1] **(**=semipreopen [5]) set if $A \subset ClIntCl(A)$.

The complement of a s-open (resp. p-open , β-open) set is called s-closed[7] (resp. p-closed [13] , β-closed [1]) set. The family of s-open (resp. p-open , β-open) sets of X is denoted by SO(X) (resp. PO(X) , βO(X)) .

Definition 2.2 : The intersection of all s-closed (resp. β-closed) sets containing a subset A of space X is called the s-closure [7] (resp. β-closure[2]) of A and is denoted by sCl(A) (resp. βCl(A)).

Definition 2.3 : The union of all s-open (resp. β-open) sets which are contained in A is called the s-interior[7] (resp. the β-interior [2]) of A and is denoted by sInt(A) (resp. βInt(A)).

Definition 2.4 [6]: A subset A of a space X is called gs-closed if $sCl(A) \subset U$ whenever $A \subset U$ and U is open set in X.

Definition 2.5[22]: A space X is said to be **β**-regular if for each closed set F and for each $x \in X-F$, there exist two disjoint **β-open** sets U and V such that $x \in U$ and $F \subset V$.

Definition 2.6 [19]: A space X is said to be **s**-regular if for each closed set F and for each $x \in X-F$, there exist two disjoint **s-open** sets U and V such that $x \in U$ and $F \subset V$.

Definition 2.7 [12]: A space X is said to be **semi**-regular if for each semiclosed set F and for each $x \in X-F$, there exist two disjoint **s-open** sets U and V such that $x \in U$ and $F \subset V$.

Definition 2.8 [18]: A space X is said to be **s**-normal if for any pair of disjoint closed subsets A and B of X , there exist disjoint **s-open** sets U and V such that $A \subset U$ and $B \subset V$.

Definition 2.9 [11]: A space X is said to be **semi**-normal if for any pair of disjoint semiclosed subsets A and B of X , there exist disjoint **s-open** sets U and V such that $A \subset U$ and $B \subset V$.

Definition 2.10 [25]: A space X is said to be submaximal if every dense set of X is open in X (i.e., every preopen set of X is open in X).

Definition 2.11[14]: A space X is said to be extremely disconnected (in brief , E.D.,) space if Cl(G) is open set for each open set G of X .

Definition 2.12[4]: PS- spaces .A space X is said to be **s**-regular [ ] if for each closed set F and for each $x \in X-F$, there exist two disjoint **s-open** sets U and V such that $x \in U$ and $F \subset V$.

Definition 2.13 [17]: A function $f: X \to Y$ is said to be **β**-irresolute if $f^{-1}(V)$ is is **β**-open set in X for each for each **β**-open set V in Y.

Definition 2.14[8] : A function $f: X \to Y$ is said to be irresolute if $f^{-1}(V)$ is is s-open set in X for each for each s-open set V in Y.

Definition 2.15[9] : A function $f: X \to Y$ is said to be presemiopen if f(V) is is semiopen set in Y for each for each semiopen set V in X.

Definition 2.16[22] : A function $f: X \to Y$ is said to be M-β-closed if the image of each β-closed set of X is β-closed in Y.

3. Properties of βs-regular spaces

We, define the following.

Definition 3.1 : A topological space X is said to be βs-regular if for each β -closed set F of X and each point x in X - F, there exist disjoint s-open sets U and V such that $x \in U$ and $F \subset V$.

Clearly, every βs-regular space is s-regular as well as semi-regular , since every closed set as well as s-closed set is β-closed set.

Definition 3.2 : A space X is said to be β*-regular if for each **β**-closed set F and for each $x \in X-F$, there exist **β**-open sets U and V such that $F \subset U$ and $x \in V$.

Theorem 3.3 : Every β*-regular space is β-regular.

Proof : Let X be β*-regular space and F be a closed set not containing x implies F be a **β**-closed set not containing x. As X is β*-regular space , there exist disjoint β-open sets U and V such that $x \in V$ and $F \subset U$. Therefore X is **β**-regular.

Lemma 3.4: If A is subset of X and $B \in \beta O(X)$ such that $A \cap B = \emptyset$ then $\beta Cl(A) \cap B = \emptyset$.

Proof : Let us assume that $\beta Cl(A) \cap B \neq \emptyset$ implies there exists x such that $x \in \beta Cl(A)$ and $x \in B$. Now $x \in \beta Cl(A) \Rightarrow A \cap U \neq \emptyset$ for each **β**-open set U containing x which is contradiction to hypothesis $A \cap B = \emptyset$ for $B \in \beta O(X,x)$.Hence $\beta Cl(A) \cap B = \emptyset$.

Lemma 3.5 : Every βs-regular space is β*-regular space .

Proof : Let X be βs-regular space and F be a **β**-closed set not containing x. As X is βs-regular space , there exist disjoint s-open **sets** U and V such that $x \in V$ and $F \subset U$. But, every s-open set is β-open set and thus X is β*-regular.

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Converse of Lemma-3.5 is not true . For example,

Example 3.6 : Let $X = \{ a,b,c\}$, $\tau = \{ X, \varnothing, \{a\}, \{b,c\} \}$

$\beta O(X) = \{ X, \varnothing, \{a\}, \{b\}, \{c\}, \{a,b\},\{a,c\},\{ b,c\} \}$

$SO(X) = \{ X, \varnothing, \{a\}, \{b,c\} \}$

Clearly X is β^*-regular space but not βs-regular . Since for β-closed set $\{a,b\}$ and $c \notin \{a,b\}$ there donot exist disjoint s-open sets U and V such that $c \in U$ and $F \subset V$.

Theorem 3.7: For a topological space X the following statements are equivalent ;

(a) X is **βs-**regular
(b) For each $x \in X$ and for each **β**-open set U containing x there exists a s-open set V containing x such that $x \in V \subset sCl(V) \subset U$.
(c) For each **β**-closed set F of X , $\cap \{sCl(V)/F \subset V$ and $V \in SO(X)\} = F$
(d) For each nonempty subset A of X and each $U \in \beta O(X)$ if $A \cap U \neq \varnothing$ then there exists $V \in SO(X)$ such that $A \cap V \neq \varnothing$ and $sCl(V) \subset U$
(e) For each nonempty subset A of X and each $F \in \beta F(X)$ if $A \cap F = \varnothing$ then there exists $V,W \in SO(X)$ such that $A \cap V \neq \varnothing$, $F \subset W$ and $V \cap W = \varnothing$.

Proof : (a) ⇒(b) Let X be **βs-** regular space. Let $x \in X$ and U be **β-**open set containing x implies X - U is **β**-closed such that $x \notin X- U$. Therefore by (a) there exists two s-open sets V and W such that $x \in V$ and $X-U \subset W \Rightarrow X-W \subset U$. Since $V \cap W = \varnothing \Rightarrow sCl(V) \cap W = \varnothing \Rightarrow sCl(V) \subset X- W \subset U$. Therefore , $x \in V \subset sCl(V) \subset U$.

(b) ⇒ (c) Let F be a **β**-closed subset of X and $x \notin F$, then X - F is **β**-open set containing x. By (b) there exists s-open set U such that $x \in U \subset sCl(U) \subset X-F$ implies $F \subset X-sCl(U) \subset X-U$ i.e $F \subset V \subset X-U$ where $V = X-sCl(U) \in SO(X)$ and $x \notin V$ that implies $x \notin sCl(V)$ implies $x \notin \cap \{ sCl(V) / F \subset V \in SO(X)\}$. Hence , $\cap \{ sCl(V) / F \subset V \in SO(X)\} = F$

(c) ⇒ (d) A be a subset of X and $U \in \beta O(X)$ such that $A \cap U \neq \varnothing$.

⇒ there exists $x_0 \in X$ such that. $x_0 \in A \cap U$. Therefore X-U is **β-**closed set not containing $x_0 \Rightarrow x_0 \notin \beta Cl(X-U)$. By (c) , there exists $W \in SO(X)$ such that $X-U \subset W \Rightarrow x_0 \notin sCl(W)$. Put $V = X-sCl(W)$, then V is s-open set containing x_0. $\Rightarrow A \cap V \neq \varnothing$ and $sCl(V) \subset sCl(X-sCl(W)) \subset sCl(X-W)$.Therefore , $sCl(V) \subset sCl(X-W) \subset U$.

(d) ⇒ (e) Let A be a nonempty subset of X and F be **β**-closed set such that $A \cap F = \varnothing$.Then X-F is **β**-open in X and $A \cap (X-F) \neq \varnothing$. Therefore by (d) , there exist $V \in SO(X)$ such that $A \cap V \neq \varnothing$ and $sCl(V) \subset X - F$. Put $W = X- sCl(V)$ then $W \in SO(X)$ such that $F \subset W$ and $W \cap V = \varnothing$.

(f) ⇒ (a) Let $x \in X$ be arbitrary and F be **β**-closed set not containing x. Let $A= X \backslash F$ be a nonempty **β**-open set containing x then by (e) ,there exist disjoint s-open sets V and W such that $F \subset W$ and $A \cap V \neq \varnothing \Rightarrow x \in V$. Thus X is a **βs-**regular.

Theorem 3.8 : In a topological space X following statements are equivalent;

(a) X is **βs**-regular
(b) for each **β**-open set U of X containing x there exists s-open set V such that $x \in V \subset sCl(V) \subset U$

Proof : (a) ⇒ (b) Let $x \in X$ and U be **β**-open set of X containing x ⇒ X - U is **β**-closed set not containing x. As X is **βs-** regular , there exist disjoint s-open sets V and W such that $x \in V$ and $X\text{-}U \subset W \Rightarrow X\text{-}W \subset U$. As $V \cap W = \emptyset \Rightarrow sCl(V) \cap W = \emptyset \Rightarrow sCl(V) \subset X\text{-}W \subset U$. Hence $x \in V \subset sCl(V) \subset U$.

(b) ⇒ (a) Let for each $x \in X$, F be **β**-closed set not containing x , therefore X-F is **β**-open set containing x hence from (b) there exists s-open set V such that $x \in V \subset sCl(V) \subset X\text{-}F$. Let U = X –sCl(V) then U is s-open set such that $F \subset U$, $x \in V$ and $U \cap V = \emptyset$. Thus there exists disjoint s-open sets U and V such that $x \in V$ and $F \subset U$. Therefore X is **βs-** regular.

We, define the following.

Definition 3.9: A space X is said to be strongly β*-regular if for each **β-c**losed set F and for each $x \in X\text{-}F$, there exists disjoint open sets U and V such that $F \subset U$ and $x \in V$.

Theorem 3.10: Every strongly β*- regular space is β*-regular space.

Proof : Let X be strongly β*-regular space and F be any β-closed set and $x \notin F$. Then there exist disjoint open sets U and V such that $x \in U$ and $F \subset V$ .Since every open set is **β**-open and hence U and V are **β-** open sets such that $x \in U$ and $F \subset V$. This shows that X is **β***-regular.

Converse of the theorem 3.10 and other statements are not true in general. For,

Example 3.11 : Let X = { a,b,c}, τ = { X, ∅, {a}, {b}, {a,b} }

βO(X) = { X, ∅, {a}, {b}, {a,b},{a,c},{ b,c} }

Clearly X is **β***-regular but not strongly **β***-regular .

Theorem 3.12 : For a topological space X the following statements are equivalent ;

(a) X is strongly **β***-regular
(b) For each $x \in X$ and for each **β**-open set U of X containing a point x , there exists an open set V such that $x \in V \subset Cl(V) \subset U$.
(c) For each **β**-closed set F of X , $\cap\{Cl(V)/F \subset V \in \tau\} = F$.
(d) For each nonempty subset A of X and each $U \in \beta O(X)$ if $A \cap U \neq \emptyset$ then there exists open set V such that $A \cap V \neq \emptyset$ and $Cl(V) \subset U$.
(e) For each nonempty subset A of X and each closed set F of X such that $A \cap F = \emptyset$, there exit , $V, W \in \beta O(X)$, such that $A \cap V \neq \emptyset$, $F \subset W$ and $V \cap W = \emptyset$.

The routine proof of the theorem is omitted.

Next , we prove some preservation theorems in the following.

Theorem 3.13 : If $f : X \to Y$ is a pre-semiopen , β-irresolute bijection and X is βs-regular space , then Y is βs-regular .

Proof : Let F be any β-closed subset of Y and $y \in Y$ with $y \notin F$. Since f is β-irresolute, $f^{-1}(F)$ is β-closed set in X. Again, f is bijective, let $f(x) = y$, then $x \notin f^{-1}(F)$.Since X is βs-regular , there exist disjoint s-open sets U and V such that $x \in U$ and $f^{-1}(F) \subset V$. Since f is , presemiopen bijection , we have $y \in f(U)$ and $F \subset f(V)$ and $f(U) \cap f(V) = f(U \cap V) = \varnothing$. Hence, Y is βs-regular space.

We , define the following.

Definition 3.14 : A function $f : X \to Y$ is said to be always-β-closed if the image of each β-closed subset of X is β-closed set in Y.

Now, we prove the following.

Theorem 3.15 : If $f : X \to Y$ is an always β-closed , irresolute injection and Y is βs-regular space , then X is βs-regular .

Proof : Let F be any β-closed set of X and $x \notin F$. Since f is an always β-closed injection, f(F) is β-closed set in Y and $f(x) \notin f(F)$. Since Y is βs-regular space and so there exist disjoint s-open sets U and V in Y such that $f(x) \in U$ and $f(F) \subset V$. By hypothesis, $f^{-1}(U)$ and $f^{-1}(V)$ are s-open sets in X with $x \in f^{-1}(U)$, $F \subset f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \varnothing$. Hence, X is βs-regular space.

We, define the following.

Definition 3.16 : A function $f : X \to Y$ is said to be βs-continuous if the inverse image of each β-open set of Y is s-open set in X.

Next, we give the following.

Theorem 3.17 : If $f : X \to Y$ is an always β-closed , βs-continuous injection and Y is β-regular space , then X is βs-regular .

4.βs-normal spaces.

We , define the following.

Definition 4.1 : A function $f : X \to Y$ is said to be βgs-continuous if for each β-closed set F of Y , f^{-1} (F) is gs-closed set in X.

It is obvious that a function $f : X \to Y$ is βgs-continuous if and only if $f^{-1}(V)$ is gs-open in X for each β-open set V of Y.

Definition 4.2: A function $f : X \to Y$ is said to be βgs-closed if for each β-closed set F of

X , f(F) is gs-closed set in Y.

We, recall the following .

Definition 4.3[23] : A function $f : X \to Y$ is said to be :

(i)pre-gs-continuous if the inverse image of each s-closed set F of Y is gs-closed in X.

(ii)pre-gs-closed if the image of each s-closed set of X is gs-closed in Y.

Clearly, (i) every pre-βgs-continuous function is pre-gs-continuous ,

(ii) every pre-βgs-closed function is pre-gs-closed , since in both cases every s-closed set is β-closed set.

We ,prove the following.

Theorem 4.4 : A surjective function $f : X \to Y$ is βgs-closed if and only if for each subset B of Y and each β-open set U of X containing $f^{-1}(B)$, there exists a gs-open set V of Y such that $B \subset V$ and $f^{-1}(V) \subset U$.

Proof : Suppose f is βgs-closed function. Let B be any subset of Y and U be any β-open set in X containing $f^{-1}(B)$. Put V = Y – f(X-U). Then , V is gs-open set in Y such that $B \subset V$ and $f^{-1}(V) \subset U$.

Conversely, let F be any β-closed set of X . Put B = Y – f(F) , then we have $f^{-1}(B) \subset X - F$ and X-F is β-open in X. There exists a gs-open set V of Y such that $B = Y - f(F) \subset V$ and $f^{-1}(V) \subset X - F$ .Therefore , we obtain that f(F) = Y-V and hence f(F) is gs-closed set in Y .This shows that f is βgs-closed function.

We,define the following.

Definition 4.5 : A function $f : X \to Y$ is said to be strongly β-closed if the image of each β-closed set of X is closed in Y.

Definition 4.5 : A function $f : X \to Y$ is said to be always gs-closed if the image of each gs-closed set of X is gs-closed in Y.

Theorem 4.6: Let $f : X \to Y$ and $g : Y \to Z$ be two functions . Then the composition function $gof : X \to Z$ is βgs-closed if f and g satisfy one of the following conditions :

(i) f is βgs-closed and g is always gs-closed .
(ii) f is strongly β-closed and g is gs-closed .

Proof : (i)Let H be any β-closed set in X and f is βgs-closed , then f(H) is gs-closed set in Y.Again , g is always gs-closed function and f(H) is gs-closed set in Y , then gof (H) is gs-closed set in Z . This shows that gof is βgs-closed function.

(ii)Let F be any β-closed set in X and f is strongly β-closed function, then f(F) is closed set in Y. Again, g is gs-closed function and f(F) is closed set in Y, then gof (F) is gs-closed set in Z. This shows that gof is βgs-closed function.

We, define the following.

Definition 4.7 : A function f : X→ Y is said to be gsβ-closed if for each gs-closed set F of

X , f(F) is β-closed set in Y.

Theorem 4.8 : Let f : X→Y and g : Y→ Z be two functions . Then ,

(i) if f is β-losed and g is βgs-closed , then gof is gs-closed .
(ii) if f is βgs-closed and g is strongly gs- closed , then gof is strongly β-closed.
(iii) if f is M-β-closed and g is βgs-closed , then gof is βgs-closed .
(iv) if f is alays gs-closed and g is gsβ-closed , then gof is gsβ-closed.

Proof : (i). Let H be a closed set in X , then f(H) be β-closed set in Y since f is β-closed function. Again , g is βgs-closed and f(H) is β-closed set in Y then g(f(H))= gof(H) is gs-closed set in Z .Thus, gof is gs-closed function.

(ii)Let F be any β-closed set in X and f is βgs-closed function, then f(F) is gs-closed set in Y. Again , g is strongly gs-closed and f(F) is gs-closed set in Y , then gof(F) is closed set in Z .This shows that gof is strongly β-closed function.

(iii)Let H be any β-closed set in X and f is M-β-closed function then f(H) is β-closed set in Y. Again, g is βgs-closed function and f(H) is β-closed set in Y , then gof (H) is gs-closed set in Z. Therefore , gof is βgs-closed function.

(iv)Let H be any gs-closed set in X and f is always gs-closed function , then f(H) be gs-closed set in Y . Again , g is gsβ-closed function and f(H) is gs-closed set in Y, then gof (H) is β-closed set in Z . This shows that gof is gsβ-closed function.

Theorem 4.9 : Let f : X→Y and g : Y→ Z be two functions and let the composition function gof : X→ Z is βgs-closed.Then, the following hold :

(i) if f is β-irresolute surjection , then g is βgs-closed.
(ii) if g is gs-irresolute injection , then f is βgs-closed .

Proof : (i) Let F be a β-closed set in Y . Since f is β-irresolute surjective , $f^{-1}(F)$ is β-closed set in X and (gof)(f^{-1}(B)) = g(F) is gs-closed set in Z . This shows that g is βgs-closed function.

(ii)Let H be a β-closed set in X .Then, gof (H) is gs-closed set in Z . Again , g is gs-irresolute injective , g^{-1}(gof(H)) = f(H) is gs-closed set in Y . This shows that f is βgs-closed function.

We , define the following.

Definition 4.10: A space X is said to be βs-normal if for any pair of disjoint β-closed sets A and B of X ,there exist disjoint s-open sets U and V such that $A \subset U$ and $B \subset V$.

Clearly, every βs-normal space is s-normal as well as semi-normal , since every closed set as well as s-closed set is β-closed set.

Definition 4.11 : A space X is said to be β^*-normal if for any pair of disjoint β-closed sets A and B of X ,there exist disjoint open sets U and V such that $A \subset U$ and $B \subset V$.

Clearly , every β^*-normal space is βs-normal space.

We , recall the following.

Lemma 4.12 [6] : A subset A of a space X is gs-open iff $F \subset sInt(A)$ whenever $F \subset A$ and F is closed set in X.

Lemma 4.13 [3] : If X is submaximal and E.D. space , then every β-open set in X is open set.

We, characterize the βs-normal spaces in the following.

Theorem 4.14 : The following statements are equivalent for a submaximal and E.D., space X :

(i) X is βs-normal space,

(ii) For any pair of disjoint β-closed sets A , B of X , there exist disjoint gs-open sets U , V such that $A \subset U$ and $B \subset V$,

(iii) For any β-closed set A and any β-open set V containing A , there exists a gs-open set U such that $A \subset U \subset sCl(U) \subset V$.

Proof : (i) $\rightarrow$ (ii) . Obvious , since every s-open set is gs-open set.

(ii)$\rightarrow$(iii) . Let A be any β-closed set and V be any β-open set containing A . Since A and X-V are disjoint β-closed sets of X , there exist gs-open sets U and W of X such that $A \subset U$ and $X-V \subset W$ and $U \cap W = \emptyset$. By Lemmas– 4.12 and 4.13 , we have $X - V \subset sInt(W)$. Since $U \cap sInt(W) = \emptyset$, we have $sCl(U) \cap sInt(W) = \emptyset$ and hence $sCl(U) \subset X - sInt(W) \subset V$. Thus , we obtain that $A \subset U \subset sCl(U) \subset V$.

(iii)$\rightarrow$ (i). Let A and B be any disjoint β-closed sets of X . Since X-B is β-open aet containing A, there exists a gs-open set G such that $A \subset G \subset sCl(G) \subset X-B$. Then by Lemmas-4.12 and 13 , $A \subset sInt(G)$. Put U = sInt (G) and V = x-sCl(G). Then , U and V are disjoint s-open sets such that $A \subset U$ and $B \subset V$. Therefore, X is βs-normal space.

Theorem 4.15 : If $f : X \rightarrow Y$ is a β-irresolute , βgs-closed surjection and X is βs-normal, then Y is βs-normal space.

Proof : Let A and B be any disjoint β-closed sets of Y. Then, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint β-closed sets of X since f is β-irresolute function . Since X is βs-normal , there exist disjoint s-open sets U and V in X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. By Th.4.4 , there exist gs-open sets G and H of Y such that $A \subset G$, $B \subset H$, $f^{-1}(G) \subset U$ and $f^{-1}(H) \subset V$. Then , we have

$f^{-1}(G) \cap f^{-1}(H) = \square$ and hence $G \cap H = \square$. It follows from Th.4.14 that space Y is βs-normal.

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