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MHD FLOW OF VISCOUS FLUID THROUGH A CIRCULAR TUBE FILLED WITH A MEDIUM OF VARYING PERMEABILITY

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Abstract : -

The flow of a viscous incompressible electrically conducting fluid in a circular tube filled with a medium of variable permeability has been considered. The permeability of the porous medium is exponentially decreasing in the radial direction. A transverse static magnetic field is applied and problem has been modeled with Brinkman Model. The modeled problem has been attempted to solved by Adomian Decomposition method. The effect of various parameters on wall shear stress, volume rate of flow and resistivity to flow has been computed and presented through the graphs.

Keywords: Magneto hydrodynamics, porous medium, Adomian Decomposition.

Introduction :- Song and Wang (2013) dealt that in this examination, we present a latest advanced Adomian decomposition procedure, that establishes a confluence- restraint parameter into the Adomian decomposition procedure and introduces a latest insistent method. It shows that the proposed procedure is authentic, valid, simple to execute from a mathematical stance. It can be engaged to acquire profitably systematic indefinite solution of fractional differential equation. Prasad and Kumar (2011) considered that through a porous medium, an analytical solution of the flow of a hydromagnetic fluid between permeable beds is acquired and analyzed. It is observed that the fluid is under an exponential decaying pressure gradient and the stable magnetic field in a direction normal to the flow saturated porous medium .

Eldesoky (2012) analized that unsteady pulsatile flow of blood through porous medium in an artery under the impact of periodic body acceleration and slip condition. With the Laplace transform, mathematical solution of the equation of motion is acquired. The mathematical definite expressions of axial velocity, wall shear stress and fluid acceleration are given.

Herzallah and Gepreel (2012) dealt that in this study we operate the adomian decomposition procedure, by establishing the fractional derivatives in the sight of Laputo, to build the indefinite solutions for the cubic non linear fractional Schordinger equation with time and space fractional derivatives. Tanveer (2016) considering blood as a couple stress, fluid, the mathematical model for steady flow of blood through a porous medium in a rigid circular tube under the influence of periodic body acceleration and magnetic field is studied. An exact solution in the Bessel's Fourier series form by the finite Hankel transform techniques, the physiological parameters that affect human body such as axial velocity shear stress and the flow rate have been computed analytically Velocity of blood decreases with increase in magnetic field, whereas increases with increase in permeability of the porous media and body acceleration. So graphically effects of shear stress and other parameters are shown.

Formulation of the problem: Let us consider a steady, viscous, axially symmetric, incompressible fluid in a circular tube of radius *R*. A static transverse magnetic field of uniform strength has been applied. Considering cylindrical polar co-ordinates(r, θ, z), where z-axis coincide with the axis of tube, then Brinkman momentum equation and boundary conditions are:

$$\mu_{eff}\left(\frac{d^2u'}{dr^2}H+\frac{1}{r}\frac{du'}{dr}\right)-\frac{\mu}{K}u'+J-\sigma B_0{}^2u'=0 \qquad \ldots(1)$$

$$r=R \quad : \quad u'=0 \qquad \text{(no slip condition)} \qquad \ldots(2)$$

$$r=0 \quad : \quad \frac{\partial u'}{\partial r}=0 \qquad \text{(symmetry condition)} \qquad \ldots(3)$$

where u', μ and σ are the axial velocity, viscosity and magnetic conductivity of the fluid, K the permeability of the porous medium, B_0 the electromagnetic induction, $J=-\frac{\partial p}{\partial z}$ the constant pressure gradient, μ_{eff} the effective viscosity.

Introducing following non dimensional parameters

$$r^*=\frac{r}{R},\qquad u^*=\frac{u'}{\left(\frac{JR^2}{\mu}\right)},\qquad \tau^*=\frac{\tau}{JR},\qquad Q^*=\frac{Q}{\left(\frac{R^4J}{\mu}\right)},\qquad \lambda^*=\frac{\lambda}{\left(\frac{\mu}{KR^2}\right)},$$

$$M=\frac{\mu eff}{\mu},\quad Da^2=\frac{K}{R^2},\qquad H^2=\frac{R^2\sigma B_0{}^2}{\mu},\qquad s=\frac{1}{Da} \qquad \ldots(4)$$

Case-I: Constant permeability (K=constant)

The equation of motion in dimensionless form is defined by

$$\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}-\left(\frac{s^2+H^2}{M}\right)u=-\frac{1}{M} \qquad \dots (5)$$

The corresponding boundary conditions are

r=1 : $u^{'}=0$

…(6)

r=0 : $\frac{\partial u^{'}}{\partial r}=0$

…(7)

Method of solution :

Employing Adomian Decomposition Method, the equation (5) reduced to

$$L_n u=-\frac{1}{M}-\frac{1}{r}\frac{du_n}{dr}+\left(\frac{s^2+H^2}{M}\right)u_n \qquad \dots(8)$$

$$L_n^{-1}=\int_0^r\int_0^r(.)\,dr^2 \qquad \dots(9)$$

Applying L_n^{-1} to both sides of the equation (8) and using the boundary condition (7), we obtain

$$u(r)=A-\frac{1}{M}\frac{r^2}{2!}-L_n^{-1}\left(\frac{1}{r}\frac{du_n}{dr}\right)+\left(\frac{s^2+H^2}{M}\right)L_n^{-1}\left(u_n\right) \qquad \dots(10)$$

where A=u(0) is to be determined from the boundary condition (6). As usual in Adomian Decomposition Method ,the solution of the equation (10) is approximated as an infinite series

$$u(r)=\sum_{i=0}^{\infty}u_i \qquad \dots (11)$$

thus, we can write equation(10) as

$$u=u_0+u_{n+1}$$

where

$$u_0=A-\frac{1}{M}\frac{r^2}{2!} \qquad \dots(12)$$

$$u_{n+1} = -L_n^{-1}\left(\frac{1}{r}\frac{du_n}{dr}\right) + \left(\frac{s^2 + H^2}{M}\right) L_n^{-1}(u_n) \qquad \ldots(13)$$

thus

$$u(r) = 2A\left\{\begin{matrix} \frac{1}{2} - \frac{r^2}{2!} + \left(\frac{10}{27}N^2 + \frac{10}{18}N^4\right)\frac{r^4}{4!} - \left(\frac{548}{450}N^4 + \frac{16}{60}N^6\right)\frac{r^6}{6!} \\ + \left(\frac{71}{105}N^6 + \frac{1}{2}N^8\right)\frac{r^8}{8!} - N^8\frac{r^{10}}{10!} \end{matrix}\right\} + O(r^{11})$$

…(14)

where $N^2 = \left(\frac{s^2 + H^2}{M}\right)$

Case-II : Variable permeability defined by

$$K = K_0 \exp(-ar) \qquad \ldots(15)$$

The equation of motion (1) reduces into

$$\mu_{eff}\left(\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr}\right) - \frac{\mu}{K_0 \exp(-ar)}u + J - \alpha B_0 u = 0 \qquad \ldots(16)$$

Invoking following dimensionless parameters

$$\frac{K_0}{R^2} = Da\ ,\ \alpha = aR\ ,\ A_1 = \frac{1}{MDa^2}\ ,\ A_2 = \frac{H^2}{M}\ ,\ A_1 + A_2 = N^2 \qquad \ldots(17)$$

alongwith the dimensionless parameters defined in (4), the equation (16) reduces to

$$\frac{d^2u^*}{dr^{*2}} + \frac{1}{r^*}\frac{du^*}{dr^*} - A_1 \exp(\alpha r)u^* + \frac{1}{M} - A_2 u^* = 0 \qquad \ldots(18)$$

for the sake of brevity , asterisks are dropped immediately

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr^2} - A_1 \exp(\alpha r)u + \frac{1}{M} - A_2 u = 0 \qquad \ldots(19)$$

Using Adomian Decomposition Method the solution of the equation (19) is obtained and given by

$$u(r) = u_0 + u_{n+1}.$$

$$u(r) = 2A\left\{\begin{array}{l}\frac{1}{2} - \frac{r^2}{2!} + \frac{1}{4}\alpha A_1 \frac{r^3}{3!} + \left(\frac{1}{3}\alpha^2 A_1 + \frac{1}{3}N^2 + \frac{1}{2}N^4\right)\frac{r^4}{4!} \\ \quad + \left(-\frac{\alpha^3 A_1}{8} + 2\alpha A_1 A_2 + \frac{15}{4}\alpha A_1 + 2\alpha A_1{}^2\right)\frac{r^5}{5!} \\ \quad\quad + \left(-\frac{\alpha^4 A_1}{10} + \frac{36}{5}\alpha^2 A_1 + \frac{11}{2}\alpha^2 A_1{}^2 - N^4 + \frac{7}{2}\alpha^2 A_1 A_2\right)\frac{r^6}{6!} + \ldots\end{array}\right\} \quad \ldots(20)$$

where

$$u_0 = 2A\left(\frac{1}{2} - \frac{r^2}{2!}\right)$$

Wall Shear Stess : The dimensionless wall shear stress is defined by

$$\tau^* = \frac{-\mu\left(\frac{\partial u}{\partial r}\right)_{r=R}}{JR} = -\left(\frac{\partial u^*}{\partial r}\right)_{r=1} \quad \ldots(24)$$

Volume rate of flow: The dimensionless volume rate of flow Q is defined by

$$Q^* = 2\pi\int_0^1 r^* u^* dr^* \quad \ldots(25)$$

Resistivity of flow: The Resistivity of the flow is given by

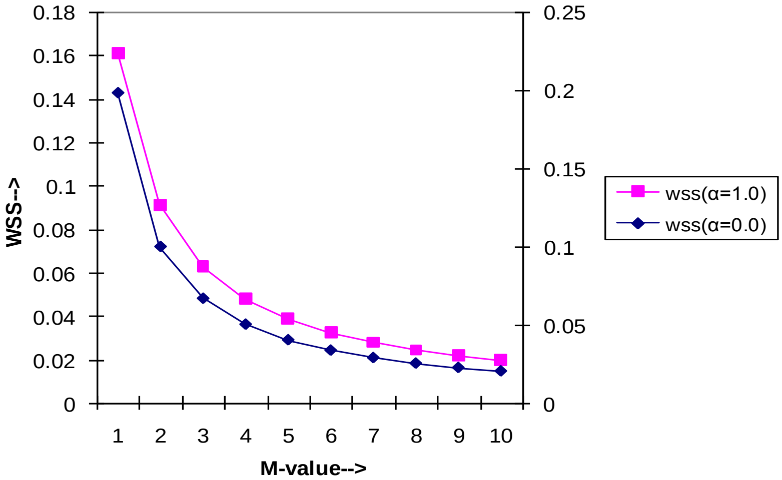
$$\lambda = \frac{\nabla p}{Q} = -\frac{\mu u}{KQ}$$

The dimensionless form is defined by

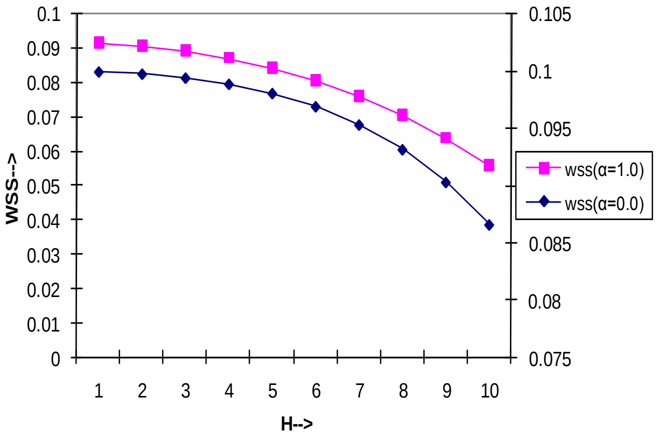
$$\lambda^* = -\frac{u^*}{Q^*} \quad \ldots(26)$$

Result:

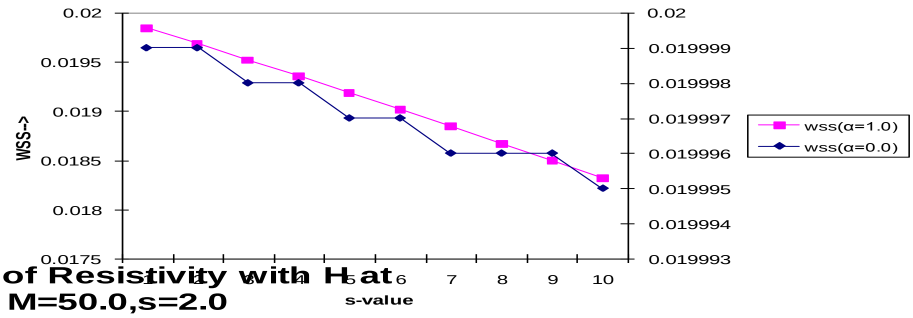
Variation of WSS with M at H=1.0,s=2.0



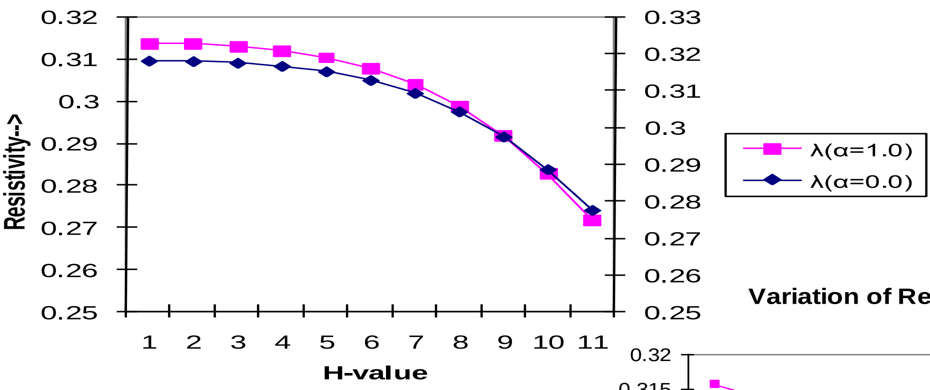
Variation of WSS with H at M=50.0,s=2.0



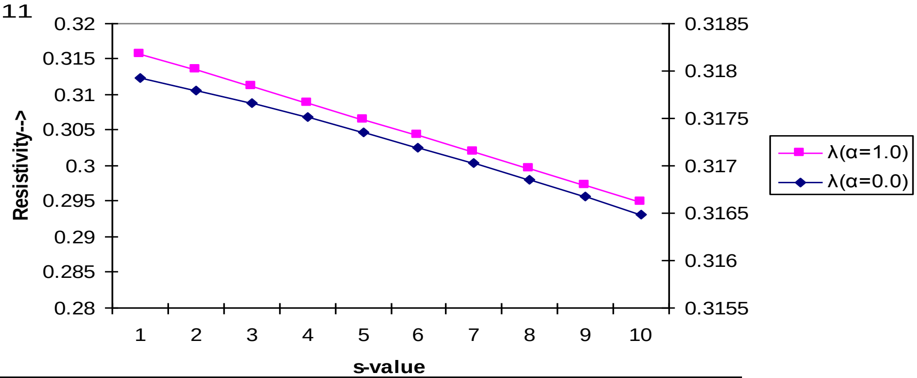
Variation of WSS with s at H=1.0,M=50.0

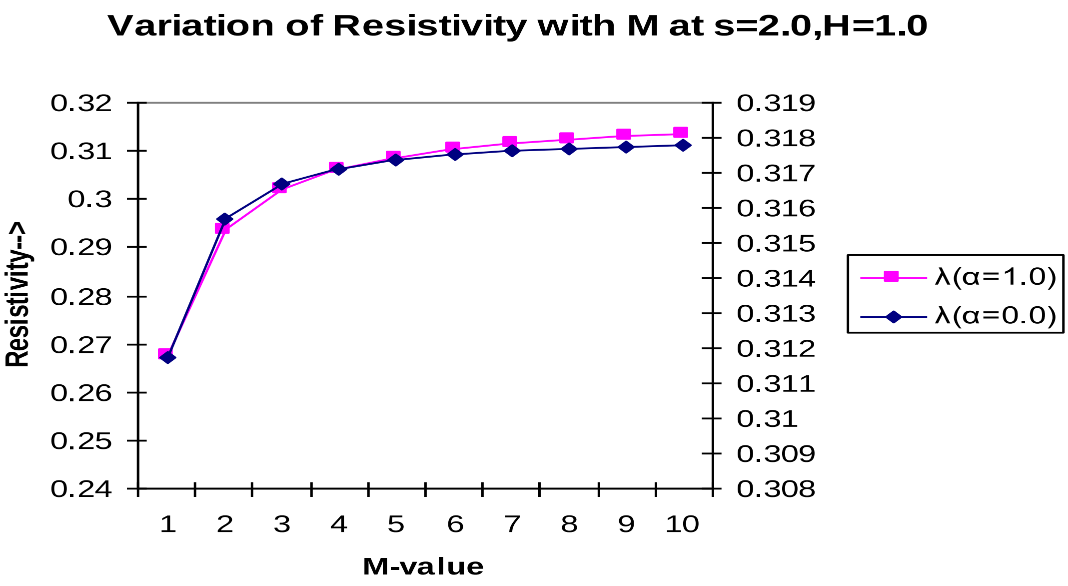


Variation of Resistivity with H at M=50.0,s=2.0



Variation of Resistivity with s at M=50.0,H=1.0





Hence there is a significant effect of non dimensional parameters on hemodynamic parameters.

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