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On Fuzzy $\gamma*$ - Semi Boundary Sets in Fuzzy Topological Spaces

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Abstract

The aim of this paper is to introduce the concept of fuzzy γ^* - semi boundary sets in fuzzy topological space. Some characterizations are discussed, examples are given and properties are established.

Key words: γ^* -semi interior, γ^* -semi closure, fuzzy γ^*-semi open and fuzzy γ^* -semi closed, γ^* -boundary sets. In this section we define γ^* -semi boundary and discuss their properties examples

INTRODUCTION

After the introduction of fuzzy sets by Zadeh [4] in 1965 and fuzzy topology by chang [1] in 1967, Several researches were conducted on generalizations of the notions of fuzzy sets and fuzzy topology. Azad introduced the notions of fuzzy semi open and fizzy semi closed sets and T.Noiri and O.R Sayed [2] introduced the notion of γ open sets and γ closed sets. In this paper we introduce fuzzy $\gamma*$ semi boundary sets and its properties are established in fuzzy topological spaces.

PRELIMINARIES

Through this paper (X, τ) denote fuzzy topological spaces. For a fuzzy set A in a fuzzy topological space$X$. The$cl(A)$, $int(A)$, A^c denote the closure, interior, complement of A respectively.

Definition:2.1 A fuzzy set A of (X, τ) is called

1. Fuzzy semi open (in short Fs open) if $A \leq cl\ (Int\ (A))$ and a fuzzy semi closed (in short Fs – closed) if $Int\ (cl\ (A)) \leq A$.

2. Fuzzy preopen (in short Fp-open if $A \leq Int\ (cl(A))$ and a fuzzy pre closed (in short Fp-closed) if $cl(Int(A)) \leq A$.

3. Fuzzy strongly semi open (in short Fss –open if $A \leq int\ (cl\ (IntA))$ and a fuzzy strongly semi closed (in short Fssclosed) $cl\ (Int\ (cl(A)) \leq A$.

4. Fuzzy γ^* - semi open if $int(A) \leq cl\ (\gamma - int\ (A))$ and fuzzy γ^* - semi closed if $cl(A) \geq int\ (\gamma - cl(A))$.

5. fuzzy γ^* -boundary of A is defined as γ^* - Bd-(A) = γ^*cl(A) $\wedge\gamma^*$cl (A^c).

Definition:2.2

Let a be a fuzzy set in the fuzzy topological space (X, τ). Then the fuzzy semi boundary of A is defined as $\gamma^* - sBd(A) = \gamma^* - sCl(A) \wedge \gamma^* - sCl\ (A^c)$

Proposition:2.3

Let A be a fuzzy set in the fuzzy topological space (X, τ) then followings are hold.

1. $\gamma^* - sBd(A) = \gamma^* - sCl\ (A^c)$

2. If A is fuzzy γ^* semi closed then $\gamma^* - sBd(A) \leq A$

3. If A is fuzzy γ^*semi open then $\gamma^* - sBd(A) \leq A^c$

Proof:

(1) By definition 1, $\gamma^* - sBd(A) = \gamma^* - sCl(A) \wedge \gamma^* - sCl\,(A^c)$ ………………. 1 $\gamma^* - sBd\,(A^c) = \gamma^* - sCl\,(A^c) \wedge \gamma^* - sCl\,(A)$ ……………... 2

From 1 and 2 we have, $\gamma* - sBd(A) = \gamma* - sBd\,(A^c)$

(2) Let A be γ^* semi closed set. (i.e.) $\gamma^* - sCl(A) = A, \gamma^* - sBd(A) = \gamma^* - sCl(A) \wedge \gamma^* - sCl\,(A^c) \leq \gamma^* sCl(A) = A$.

(3) Let A be γ^* semi open set, (i.e.)., $\gamma^* - sint(A) = A, \gamma^* - sBd(A) = \gamma^* - sCl(A) \wedge \gamma^* sCl\,(A^c) \leq \gamma^* sCl\,(A^c) = [\gamma^* - sint\,(A)]^c = A^c$.

Proposition: 2.4

For a set A in the fuzzy topological space (X, τ) then followings are hold.

1. $[\gamma^* - sBd\,(A)]\, c = \gamma^* - int(A) \vee \gamma^* - sint\,(A^c)$
2. $\gamma^* - sBd(A) \leq Bd(A)$
3. $\gamma^* sCl\,(\gamma^* sBd(A)) \leq Bd(A)$

Proof:

(1) Let A be any set in fuzzy topological space, $\gamma^* - sBd(A) = \gamma^* sCl(A) \wedge \gamma^* sCl\,(A^c)$

Taking complement on both sides we get, $[\gamma^* - sBd(A)]^c$

$= [\gamma^* - sCl(A) \wedge \gamma* sCl\,(A^c\,)]^c$

$= [\gamma^* - sCl(A)]^c \vee [\gamma^* sCl(A^c)]^c = \gamma^* sint(A^c) \vee \gamma^* sint(A)$.

(2) Since $\gamma^* - scl(A) \leq cl(A)$ and $\gamma^* scl(A^c) \leq cl(A^c)$, We have, $\gamma^* - sBd(A) =$

$\gamma^* scl(A) \wedge \gamma^* scl(A^c) \leq cl(A) \wedge cl(A^c) = Bd(A)$.

(2) Let $\gamma^* scl(\gamma^* sBd(A)) = \gamma^* - scl(\gamma^* scl(A) \wedge \gamma^* scl(A^c)) \leq \gamma^* scl(\gamma^* scl(A)) \wedge \gamma^* scl(scl(A^c)) = \gamma^* scl(A) \wedge \gamma^* scl(A^c) = \gamma^* sBd(A) \leq Bd(A)$ Thus, $\gamma^* scl(\gamma^* sBd(A)) \leq Bd(A)$.

Example:2.5

let A= {a.6, b.9} and B= {a.4, b.1} in the fuzzy topological space (X, τ) then $\gamma^* Bd(A) = \{a.4, b.3\} \leq A$ but A is not fuzzy γ^* semi closed and $\gamma^* Bd(B) = \{a.4, b.3\} \leq Bc$, But B is not fuzzy semi open.

Proposition: 2.6

If A be fuzzy set in the fuzzy topological space (X, τ) then, 1. $\gamma^* - sBd(A) = \gamma^* scl(A) \wedge (\gamma^* sint(A))\, c$ 2. $\gamma^* - sBd(\gamma^* - sint(A)) \leq \gamma^* - sBd(A)$.

Proof: (1)

Since $\gamma^* scl(A^c) = (\gamma^* sint(A))^c$ we have $\gamma^* - sBd(A) = \gamma^* scl(A) \wedge \gamma^* scl(A)^c = \gamma^* scl(A) \wedge (\gamma^* sint(A))^c$ (2) $\gamma^* - sBd(\gamma^* - sint(A)) = \gamma^* scl(\gamma^* sint(A)) \wedge \gamma^* scl(\gamma^* sint(A))^c = \gamma^* scl(\gamma^* sint(A)) \wedge \gamma^* - scl(\gamma^* scl(A^c)) = \gamma^* scl(\gamma^* sint(A)) \wedge \gamma^* scl(A)^c = \gamma^* scl(\gamma^* sint(A)) \wedge (\gamma^* sint(A))^c \leq \gamma^* scl(A) \wedge (\gamma^* sint(A))\, c = \gamma^* - sBd(A)$

Proposition: 2.7

Let A be fuzzy set in the fuzzy topological space (X, τ) then,

1. $\gamma^* sBd(\gamma^* scl(A)) \leq \gamma^* sBd(A)$

2. $\gamma^* sint(A) \leq A \wedge (\gamma^* - sBd\ (A))^c$

Proof:

(1) $\gamma^* - sBd(\ \gamma^* - scl(A)) = \gamma^* - scl\ (\ \gamma^* - scl(A)) \wedge \gamma^* scl(\gamma^* scl(A))^{\ c} = \gamma^* scl\ (\ \gamma^* scl(A)) \wedge (\gamma^* sint(\gamma^* scl(A)))^{\ c} \leq \gamma^* scl(A) \wedge (\gamma^* - sint(A))^{\ c} = \gamma^* sBd(A)$

(2) $A \wedge\ (\gamma^* sBd(A))^{\ c} = A \wedge\ (\gamma^* - scl(A)\ \wedge \gamma^* - scl(A^c\ ))^{\ c} = A \wedge\ (\gamma^* sint(Ac\ ) \vee \gamma^* - sint(A)) = (A \wedge\ (\gamma^* sint(A^c\ )) \vee (A \wedge \gamma^* sint(A)) = (A \wedge\ (\gamma^* sint(A^c\ )) \vee (\gamma^* sint(A)) \geq \gamma^* - sint(A)$

Example:2.8 The above inequalities (1) and (2) of proposition 4 are in irreversible. The following example choose A = {a.4, b.1} in the fuzzy topological space (X, τ) then $\gamma^* - sint(A) = \{a.\ 1, b.\ 1\}$ and $\gamma^* sBd(A) = \{a.\ 4, b.\ 3\} \nleq \gamma^* sBd\ (\gamma^* sint(A)) = \{a.\ 2, b.\ 1\}$

Example:2.9 The following shows that $A \wedge\ (\gamma^* sBd(A))^c \nleq \gamma^* sint(A)$. Let X = {a, b}, τ ={{0, 1, a.8, b.8 }, { a.2, b.2 }, { a.3, b.7 }} then (x, τ) is a fuzzy topological space the closed sets τc = {0, 1, { a.2, b.2 }, { a.8, b.8 }, { a.7, b.3 }}.

Let A= {a.4, b.1} then $\gamma^* - sint(A) = \{a.\ 2, b.\ 0\}$ and $\gamma^* - sBd(A) = \{a.\ 4, b.\ 3\}$. It follows that $A \wedge\ (\gamma^* sBd(A))^{\ c} = \{a.\ 4, b.\ 1\} \nleq \gamma^* sint\ (A)$.

Theorem:2.10

Let A and B be a fuzzy set in the fuzzy topological space (X, τ) then, $\gamma^* sBd$ (A ∨B) ≤ $\gamma^* sBd(A) \vee \gamma^* sBd$(B). Proof: Let A and B be a fuzzy set in the fuzzy topological space (X, τ) , then $\gamma^* sBd$(A ∨ B) = $\gamma^* scl(A \vee$ B) $\wedge \gamma^* scl(A \vee$ B)c = $\gamma^* scl(A \vee$ B) $\wedge \gamma^* scl(A^c \wedge B^c\ ) \leq (\gamma^* scl(A) \vee \gamma^* scl$(B)) $\wedge$ $(\gamma^* scl(Ac\ ) \wedge \gamma^* scl(Bc\ )) \leq (\gamma^* scl(A) \wedge \gamma^* scl(Ac\ )) \vee$ $(\gamma^* scl$(B) $\wedge \gamma^* scl(Bc\ )) = \gamma^* sBd(A) \vee (\gamma^* sBd$(B))

Theorem:2.11

Let A and B are any two fuzzy sets in the fuzzy topological space (X, τ) then,

$\gamma^* sBd(A \wedge B) \leq (\gamma^* sBd(A) \wedge \gamma^* scl(B)) \vee (\gamma^* sBd(B) \wedge \gamma^* scl(A))$ Proof: By known the Lemma, $\gamma^* sBd(A \wedge B) = \gamma^* scl(A \wedge B) \wedge \gamma^* scl(A \wedge B)^c = \gamma^* scl(A \wedge B) \wedge \gamma^* scl(A^c \vee B^c) \leq (\gamma^* scl(A) \wedge \gamma^* scl(B)) \wedge (\gamma^* scl(A^c)) \vee (\gamma^* scl(B^c)) = (\gamma^* scl(A) \wedge \gamma^* scl(B) \wedge \gamma^* scl(A^c)) \vee (\gamma^* scl(A) \wedge \gamma^* scl(B) \wedge \gamma^* scl(B^c)) = (\gamma^* sBd(A) \wedge \gamma^* scl(B)) \vee (\gamma^* sBd(B) \wedge \gamma^* scl(A))$

Theorem:2.12

Let (X, τ) be a fuzzy topological space. If A is a fuzzy subset of a fuzzy topological space X and B is a fuzzy subset of a fuzzy topological space Y, then

1. $\gamma^* scl(A) \times \gamma^* scl(B) \geq \gamma^* scl\ (A \times B)$

2. $\gamma^* sint(A) \times \gamma^* sint(B) \leq \gamma^* sint(A \times B)$.

Proof:

Using Definition of product , $(\gamma^* scl(A) \times \gamma^* scl(B))(x, y) = min\{sclA(x), \gamma^* sclB(y)\} \geq \min\ \{A(x), B(x)\} = (A \times B)(x, y)$ This shows that, $(\gamma^* scl(A) \times \gamma^* scl(B))\ (x, y) = (x, y)$

By known Lemma, $\gamma^* scl\ (A \times B) \leq \gamma^* scl\ (\gamma^* scl(A) \times \gamma^* scl(B)) = \gamma^* scl(A) \times \gamma^* scl(B)$

By using Definition, $(\gamma^* sint(A) \times \gamma^* sint(B))\ (x, y) = min\ \{\gamma^* sintA(x), \gamma^* sintB(y)\} \geq \min\ \{A(x), B(x)\} = (A \times B)\ (x, y)$

This shows that, $(\gamma^{*}scl(A) \times \gamma^{*}scl(B)) \leq (x, y)$, By known Lemma, $\gamma^{*}sint\ (A \times B) \leq \gamma^{*}sint\ (\gamma^{*}sint(A) \times \gamma^{*}sint(B))$ $= \gamma^{*}sint(A) \times \gamma^{*}sint(B)$.

CONCLUSION

Fuzzy γ- closed set and fuzzy γ- open set are the major role in fuzzy topology, since its inception several weak forms of fuzzy γ-closed sets and fuzzy γ- open sets have been introduced in general fuzzy topology. The present paper is investigated in the new weak forms fuzzy $\gamma*$ - semi Boundary set in fuzzy topological spaces. Hence the propositions and theorems are justifying the results. We hope that the findings in this paper will help researcher enhance and promote the further study on general fuzzy topology to carry out a general framework for their applications in practical life. This paper, not only can form the theoretical basis for further applications of fuzzy topology, but also lead to the development of information systems.

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