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A STUDY ON THE FORMULATION OF LINEAR ALGEBRA

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ABSTRACT:

This research aims to design a Linear Algebra learning material that can facilitate the enhancing of students' mathematical understanding and representation. The research method is research and development (R&D) which consists of three main stage, namely the preliminary, development, and dissemination. The research was limited to the development stage. The results concluded that the assessment of the experts (validator) on learning materials is in the category of valid with a small revision of the exercise questions part. The result of a practical test of learning material is 87,81% (very practices). Meanwhile, the result of limited trials indicates that learning material can be completed students' mathematical understanding classically and individually. Besides, the mathematical representation of student has not reached the mastery both in classically and individual.

Keywords*: Linear Algebra, development, mathematical*

Introduction

The aim of giving mathematics at every level of education is to train students as a problem solver. Certain cognitive abilities need to be possessed by a problem solver so that he can maximize the knowledge he has acquired and can use it optimally. A problem solver must have the cognitive abilities needed to understand and represent a mathematical situation, create algorithms on certain problems, process various types of information, and run computing, and must also be able to identify and manage a set of appropriate resolution strategies to solve problems [1]. The ability of students to understand the prerequisite concepts and mathematical concepts being studied is the key to success in obtaining certain mathematical knowledge and solving problems encountered during learning. When students solve mathematical problems, students indirectly adapt and expand existing knowledge by connecting or linking new information obtained with prior knowledge.

A series of mental activities will make new information related to the structure of student knowledge. Students who have good problem-solving abilities are also supported by their ability to manipulate verbal language, graphics, images, and mathematical symbols [2]. The ability of mathematical representation is needed by students when faced with a problem. The representative ability can help students in simplifying a mathematical concept that is presented in several forms of representation [3]. Appropriate representation can lead students to obtain the right knowledge, while the wrong representation will lead to ill problem-solving process. Therefore, the ability of mathematical understanding and representation together is important to be possessed by students to achieve better quality learning including Linear Algebra lectures. The idea of students' mathematical understanding and representation abilities in learning has been the topic of several studies lately. The study conducted by Fatqurhohman resulted in the finding that training students to do procedural understanding will strengthen and develop their understanding of concepts [4]. Through the use of algorithms in identifying problems and connecting concepts with various representations can facilitate educators to achieve the development of the ability to understand the concept of students during learning, even though the level of understanding of students varies. Activities during learning are very important in building an understanding of both procedural and conceptual .

Another study about efforts to develop mathematical understanding ability through the design of instructional materials concluded that based on the results of the limited test of teaching materials for a group of mathematics education students, the results of the students' mathematical comprehension were both classical and individual [6]. The mathematical representation was studied by several previous researchers associated with mathematical topics at the school and college level. Most students are weak in carrying out mathematical representation activities regarding connecting procedures and processes to various representations of relevant concepts seen when students are faced with one of the questions related to Linear Algebra material [7]. The availability of teaching materials based on certain mathematical abilities would facilitate the development of mathematical abilities that are the goal of learning by paying attention to the level of diversity of students' initial abilities [8].

Literature review:

When it comes to systems of linear equations, there is not much research pertaining to what methods students tend to use, especially when multiple choice answers are involved. There is research on linear equations and research on multiple choice answers, but combining the two together and going one step further to make linear equations into systems of linear equations has not been well established (Anderson, 1989; Coppedge& Hanna, 1971; Hewitt, 2012; Huntley, Marcus, Kahan, & Miller, 2007; Kazemi, 2002; Marshall, 1983; Nogueira de Lima & Tall, 2008).

Over the years, many methods have been devised to solve system of linear equations for two variables and more than two variables.

Cornelius Lanczos [1952] A simple algorithm is described which is well adapted to the effective solution of large systems of linear algebraic equations by a succession of well-convergent approximations

FALL [2012]. This descriptive study focuses on the approaches college students (ages 20 - 24) use when solving systems of linear equations problems that have multiple choice answers. Participants were from a midsize public university in the northeast. Four approaches were considered – three forwards approaches: 1) substitution, 2) elimination, and 3) graphing, and one backwards approach: plugging in the x and y values from each multiple choice option. Participants solved systems of linear equations problems and answered questions based on their methods in a structured clinical interview

KHAN et al. [2015]. This paper comprises of matrix introduction, and the direct methods for linear equations. The goal of this research was to analyze different elimination techniques of linear equations and measure the performance of Guassian elimination and Guass Jordan method, in order to find their relative importance and advantage in the field of symbolic and numeric computation. The purpose of this research is to revise an introductory concept of linear equations, matrix theory and forms of Guassian elimination through which the performance of Guass Jordan and Guassian elimination can be measured.

Riga [2016].The systems of linear equations are a classic section of numerical methods which was already known BC. It reached its highest peak around 1600-1700 due to the public demand for solutions of technical and engineering tasks, nevertheless, it is still topical nowadays. This paper describes another iterative approach to solving linear systems, which is based in the multiple transfers of the solution proximity point towards the solution itself, simultaneously reducing the differences of all the system equations

MARYA et.al [2017]. In this paper we present a survey of three direct methods for the solution of system of linear equations. Various methods are introduced to solve systems of linear equations but in contrast to all methods not a single method are best for all situations (to get appropriate solution). These methods depend on speed and accuracy as these are an important factor in solving large systems of equations because the volume of computations involved for solution are bulky. Direct solution of simultaneous linear equations may be regarded as a slow process for large systems of equations and requires special treatment to avoid numerical instability.

.Maharaja [2018]. This paper focused on the written work of two students to questions based on the solution of a system of linear equations using matrix methods. The objective was to gauge the possible level of mathematical understanding of the students by using a framework that was arrived at. That framework was used to analyse the level of mathematical understanding of those students' written responses to the questions, with the focus on their use of symbolic language. It was found that the framework enabled the researcher to get a deeper insight into those students use of the symbolic language, used both in an instrumental role and also as a communicative function.

Objective of the study:

1. To study on Linear systems are systems of equations
2. To study on Methods That Are Used To Solve System Of Linear Equations

DEFINITION:

A system of linear equations is a collection of one or more linear equations involving the same set of variables. Or Linear systems are systems of equations in which the variables are never multiplied with each other but only the constants and then summed up.

A linear equation in variables $a_1x_1 + a_2x_2 + a_3x_3 + \cdots .. a_nx_n$=b

Where $a_{1,} a_2 , a_3 \ldots\ldots\ldots\ldots a_n$ and b are constants of real and complex numbers. The constant a_i

is called the coefficient of x_i and b is called the constant term of the equation.

Methods That Are Used To Solve System Of Linear Equations:

There are so many methods are used to solve the linear system of equation. Some of these are:- elimination of variables, Row reduction(Gauss elimination method), Cramer's rule,and etc..

ELIMINATION OF VARIABLES:

The simplest method for solving a system of linear equations is to repeatedly eliminate variables. This method can be described as follows:

- In the first equation, solve for one of the variables in terms of the others.
- Substitute this expression into the remaining equations. This yield a system of equations with one fewer unknown.

- Repeat until the system is reduced to a single linear equation. Solve this equation, and then back substitute until entire solution is found

Row Reduction Method:

To perform row reduction in a matrix, one uses a sequence of elementary row operations to modify the matrix until the lower half hand corner of the matrix is filled with zeros, as much as possible.

For example:- solve the system of equation using Row reduction method

$$x + 4y + 9z = 16$$

$$2x + y + z = 10$$

$$3x + 2y + 3z = 18$$

write it as AX=B

$$\begin{bmatrix} 1 & 4 & 9 \\ 2 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 16 \\ 10 \\ 18 \end{pmatrix}$$

Augmented matrix ;

$$\begin{bmatrix} 1 & 4 & 9 & :16 \\ 2 & 1 & 1 & :10 \\ 3 & 2 & 3 & :18 \end{bmatrix}$$

Apply $R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow R_3 - 3R_1$

Apply $R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 4 & 9 & :16 \\ 0 & -7 & -17 & :-32 \\ 0 & -10 & -24 & :-30 \end{bmatrix}$$

$$R_3 \rightarrow 7R_3 - 10R_2$$

$$\begin{bmatrix} 1 & 4 & 16 & :16 \\ 0 & -7 & -17 & :-22 \\ 0 & 0 & 2 & :10 \end{bmatrix}$$

x+4y +9z = 16

-7y -17z =-22

2z = 10

Z = 5 , y =- 9 and x = 7

5.3 Same question solved by guass Jordan method:

x + 4y + 9z = 16

2x + y + z = 10

3x + 2y + 3z = 18

write it as AX=B

$$\begin{bmatrix} 1 & 4 & 9 \\ 2 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 16 \\ 10 \\ 18 \end{pmatrix}$$

Augmented matrix $[AA : BB]$

$$\begin{bmatrix} 1 & 4 & 9 & :16 \\ 2 & 1 & 1 & :10 \\ 3 & 2 & 3 & :18 \end{bmatrix}$$

Apply $R_2 \to R_2 - 2R_1 \quad R_3 \to R_3 - 3R_1$

Apply $R_2 \to R_2 - 2R_1 \quad ,R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 4 & 9 & :16 \\ 0 & -7 & -17 & :-32 \\ 0 & -10 & -24 & :-30 \end{bmatrix}$$

$$R_3 \to 7R_3 - 10R_2$$

$$\begin{bmatrix} 1 & 4 & 16 & :16 \\ 0 & -7 & -17 & :-22 \\ 0 & 0 & 2 & :10 \end{bmatrix} R_3 \to 7R_3 - 10R_2 \ , R_1 \to 7R_1 + 4R_2$$

$$\begin{bmatrix} 7 & 0 & -5 & :24 \\ 0 & -7 & -17 & :-22 \\ 0 & 0 & 2 & :10 \end{bmatrix} R_1 \to 2R_1 + 5R_3 \quad R_2 \to 2R_2 + 17R_3 \quad ,$$

$$\begin{bmatrix} 14 & 0 & 0 & :98 \\ 0 & -14 & 0 & :126 \\ 0 & 0 & 2 & :10 \end{bmatrix}$$

-14y= 126

2z = 10

So, x=7 , y= -9 , and z = 5

5.5 Same question solved by crammers rule :

$x \ + 4y + 9z = 16$ **(1)**

$2x + \ y \ + \ z = 10$ **(2)**

$3x + 2y \ + 3z = 18$ **(3)**

First we need to find the value of delta Δ

$$\Delta = \begin{vmatrix} 1 & 4 & 9 \\ 2 & 1 & 1 \\ 3 & 2 & 3 \end{vmatrix}$$

Now we have to apply 3rd row determinant rule

$$1\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} - 4\begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix} + 9\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$$

$(3-2)-4(6-3)+9(2-3)$

1-12 +9

10– 12 = -2 so $\Delta = -2$

Now for finding the value of X,Y and Z we need to find the value of Δxx , $\Delta yy\ aaaaaa\ \Delta zz$ In Δ 1st column we have to replace by these three exponent

$$\Delta x \ = \ \begin{vmatrix} 16 & 4 & 9 \\ 10 & 1 & 1 \\ 18 & 2 & 3 \end{vmatrix}$$

$16(3-2)-4(30-18)+9(20-18)$

16-48+18 = -14

So $x = \frac{\Delta x}{\Delta}\frac{-14}{-2} = 7\ i.e\ x = 7$

$$\Delta y \ = \ \begin{vmatrix} 1 & 16 & 9 \\ 2 & 10 & 1 \\ 3 & 18 & 3 \end{vmatrix}$$

$1(30-18)-16(6-3)+9(36-30)$

12 - 48 + 54 = 18

$$y = \frac{\Delta y}{\Delta} = \frac{18}{-2} = -9$$

$$\Delta z \ = \ \begin{vmatrix} 1 & 4 & 16 \\ 3 & 1 & 10 \\ 3 & 2 & 18 \end{vmatrix}$$

$1(18-20)-4(36-30)+16(4-3)$

-2 - 24 +16 = -10

$$z = \frac{\Delta z}{\Delta} = \frac{-10}{-2} = 5$$

So, the same question solved by these methods we get the same values of x, y and z

Similarly we can solve these type of question by using different –different methods of system of linear equation very easily

Health Care Professional:

The health care field, including doctors and nurses, often use linear equations to calculate medical doses. Linear equations are also used to determine how different medications may interact with each other and how to determine correct dosage amounts to prevent overdose with patients using multiple medications. Doctors also use linear equations to calculate doses based on a patient's weight.

CONCLUSION:

This study is development of Linear Algebra learning material research. The learning material is in a valid category with a small revision on the part of the exercise questions. The result of the practice of learning materials is very practical. Meanwhile, the limited trial results show that learning materials can solve students' mathematical understanding both in classical and individual. The ability of mathematical representation of students has not reached mastery in classical or individual.

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