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TRANSIENT THERMOELASTICITY IN SQUARE PLATE

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ABSTRACT :
This paper is concerned with the analytical study of the quasi_static thermoelastic problem of the square plate occupying region space$D : 0 \le x \le a,\ 0 \le y \le a,\ 0\ \le z \le h$ .Initially Square Plate is kept at zero temperature. The outer boundaries $x = 0$ *and* $x = a$ are kept at zero prescribed temperature and thermally insulated at $y = 0$ *and* $y = a$. Temperature $\frac{-Q}{\lambda}\chi_1(x,\ y,t)$ is prescribed over $z = 0$ and $\chi_2(x,\ y,t)$ prescribed over $z = h$. Integral transform and Laplace transform techniques are used to determine the solution to the problem. Numerical calculations are carried out for the particular case of the Square Plate made up of copper (pure) metal by assigning suitable values to the parameters and functions in the equations and results are explained graphically.

KEYWORDS*: Unsteady State, thermoelastic Problem, Thermal Stresses, Fourier Sine Transform, Fourier Cosine Transform, Laplace Transform.*

INTRODUCTION Deshmukh K. C. , Quazi Y. I., Warbhe S. D. , Thermal stresses induced by a point heat source in a hollow disk by quasi-static approach Khobragade and Wankhede [4] have studied the inverse steady state thermoelastic problem to determine the temperature displacement function and thermal stresses at the boundary of a thin rectangular plate. They have used the finite Fourier sine transform technique. Dange and Khobragade [2] have studied three dimensional inverse steady-state thermoelastic problem of a thin rectangular plate. Deshmukh K. C. , Quazi Y. I., Warbhe S. D. , [3] have studied thermal stresses induced by a point heat source in a hollow disk by a quasi-static approach. Lamba and Khobragade [5] have studied thermoelastic problem of a thin rectangular plate due to a partially distributed heat supply. They have used the Marchi – Fasulo transform and Laplace transform technique. In the present paper an attempt is made to determine temperature distribution, unknown temperature at any point of a square plate, thermoelastic displacement function , displacement components and thermal stresses of square plate occupying the space$D :\ 0 \le x \le a, 0 \le y \le a, 0\ \le z \le h$, with known boundary conditions by applying finite Fourier sine transform and Fourier cosine transform and Laplace transform techniques. Numerical calculations are carried out for a particular case of square plate made of copper (pure) metal by assigning suitable values to the parameters and functions in the equations and results are depicted graphically.

STATEMENT OF THE PROBLEM :

Consider a square plate occupying the space$D : 0 \le x \le a, 0 \le y \le a, 0\ \le z \le h$. The displacement components $u_x,\ \ u_y$ and u_z in the $X,\ Y,\ Z$direction respectively are in the integral form as

$$u_x = \int_0^a \frac{1}{E}\left(\nabla^2 U - (1+\nu)U_{,xx} + \ \alpha T\right)\ dx \tag{1}$$

$$u_y = \int_0^a \frac{1}{E}\left(\nabla^2 U - (1+\nu)U_{,yy} + \ \alpha T\ \right)\ dy \tag{2}$$

$$u_z = \int_0^h \frac{1}{E}\left(\nabla^2 U - (1+\nu)U_{,zz} + \ \alpha T\right)\ dz \tag{3}$$

Where $E,\ \nu$ and α are the Young's modulus, Poison's ratio and the linear coefficient of thermal expansion of the material of the plate respectively, $\cup\,(x,\ y,\ z,\ t)$ is the Airy's stress function which satisfies the differential equation namely,

$$\nabla^4 \cup (x,\ y,\ z,\ t) = -\alpha E \nabla^2 T(x,\ y, z, t) \tag{4}$$

Where $T(x,\ y,\ z,\ t)$ denotes the temperature of the square plate satisfying the following differential equation.

$$\nabla^2 \boldsymbol{T} \ = \ k^{-1} T_{,t} \tag{5}$$

Where, k is the thermal diffusivity of the material.

The initial condition is

$$\mho_t(T, 1{,}0{,}0) = 0 \tag{6}$$

The boundary conditions are

$$\mho_x(T, 1{,}0{,}0) = 0 \tag{7}$$

$$\mho_x(T, 1{,}0, a) = 0 \tag{8}$$

$$\mho_y(T, 0{,}1{,}0) = 0 \tag{9}$$

$$\mho_y(T, 0{,}1, a) = 0 \tag{10}$$

$$\mho_z(T, 1{,}0{,}0) = \frac{-Q}{\lambda} \chi_1(x,\ y, t) \tag{11}$$

$$\mho_z(T, 1{,}0, h) = \chi_2(x, y, t) \tag{12}$$

The interior condition

$$\mho_z(T, 1{,}0, \xi) = \chi_3(x, y, t) \tag{13}$$

The stress components in terms are given by

$$\sigma_{xx} = \nabla^2 U - U_{,xx} \tag{14}$$

$$\sigma_{yy} = \nabla^2 U - U_{,yy} \tag{15}$$

$$\sigma_{zz} = \nabla^2 U - U_{,zz} \tag{16}$$

SOLUTION OF THE PROBLEM :

Applying Fourier sine transform over x to the equation (5) (7)and (8). Applying Fourier Cosine transform over y variables to the equations (9) (10), taking Laplace transform and then their inverses one obtains the expression for temperature and unknown temperature gradient $\chi_3(x, y, t)$ as

$$T(x, y, z, t) = \frac{-8k\pi}{h^2 a^2} \sum_{l,m,n=1}^{\infty} [l(-1)^{-l}] \left\{ \left[\frac{Q}{\lambda} sin \frac{l\pi}{h} (z-h) \int_0^t \overline{\chi_1}\, (m, n, t') \exp[-k\eta^2 (t - t')]\, dt' \right] - \left[sin \left(\frac{l\pi}{h} \right) z \int_0^t \overline{\chi_2}\, (m, n, t') \exp[-k\eta^2 (t-t')]\, dt' \right] \right\} sin \frac{m\pi x}{a} cos \frac{n\pi y}{a} \tag{17}$$

$$\chi_3(x,\ y,\ \ t) = \frac{8k\pi}{h^2 a^2} \sum_{,m,n,l=1}^{\infty} \left[\frac{l}{cos\, l\pi} \right] \left\{ \left[\frac{Q}{\lambda} sin \frac{l\pi}{h} (\xi - h) \int_0^t \overline{\chi_1}\, (m, n, t') \exp[-k\eta^2 (t - t')]\, dt' \right] - \left[sin \left(\frac{l\pi}{h} \right) \xi \int_0^t \overline{\chi_2}\, (m, n, t') \exp[-k\eta^2 (t - t')]\ \ dt' \ \right] \right\} sin \frac{m\pi x}{a} cos \frac{n\pi y}{a} \tag{18}$$

Where $\eta^2 \ = \ \frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{a^2} + \frac{l^2\pi^2}{h^2}$

DETERMINATION OF THERMOELASTIC DISPLACEMENT FUNCTION:

Substituting the values of $T(x,\ y,\ z, t)$ from equation (17) in equation (4) one obtains

$$U(x, y, z, t) = \frac{8\alpha k\pi E}{h^2 a^2} \sum_{,m,n,l=1}^{\infty} [l(-1)^{-l}] \left\{ \left[\frac{Q}{\lambda} sin \frac{l\pi}{h} (z-h) \int_0^t \overline{\chi_1}\, (m, n, t') \exp[-k\eta^2 (t - t')]\ \ dt' \ \right] - \left[sin \frac{l\pi}{h} z \int_0^t \overline{\chi_2}\, (m, n, t') \exp[-k\eta^2 (t - t')]\ \ dt' \ \right] \right\} sin \frac{m\pi x}{a} cos \frac{n\pi y}{a} \tag{19}$$

DETERMINATIONS OF DISPLACEMENT COMPONENTS :

Substituting the values (19) in the equation (1) to (3) one obtains

$$u_x = \frac{8\alpha k\pi}{h^2a^2}\sum_{m,n,l=1}^{\infty}\left[\frac{l(-1)^{-l}(1+\nu)\frac{m^2\pi^2}{a^2}[(-1)^{m+1}+1]}{\eta^2}\right]\left\{\left[\frac{Q}{\lambda}\sin\frac{l\pi}{h}(-h)\int_0^t\overline{\chi_1}(m,n,t')\exp[-k\eta^2(t-t')]\ dt'\right]-\left[\sin\left(\frac{l\pi}{h}\right)z\int_0^t\overline{\chi_2}(m,n,t')\exp[-k\eta^2(t-t')]\ dt'\ \right]\right\}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{a} \tag{20}$$

$$u_y = 0 \tag{21}$$

$$u_z = \frac{8\alpha k\pi}{h^2a^2}\sum_{l,m,n=1}^{\infty}l(-1)^{-l}\left[\frac{(1+\nu)\frac{l^2\pi^2}{h^2}}{\eta^2}\right]\left\{\frac{Q}{\lambda}[(-1)^l-1]\int_0^t\overline{\chi_1}(m,n,t')\exp[-k\eta^2(t-t')]\ dt' - [(-1)^{l+1}+1]\int_0^t\overline{\chi_2}(m,n,t')\exp[-k\eta^2(t-t')]\ dt'\right\}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{a} \tag{22}$$

DETERMINATION OF STRESS FUNCTION:

Substituting values of (19) in equations (14) to (16) one obtains

$$\sigma_{xx} = \frac{-8\alpha k\pi E}{h^2a^2}\sum_{l,m,n=1}^{\infty}l(-1)^{-l}\left[\frac{\eta^2-\frac{m^2\pi^2}{a^2}}{\eta^2\cos l\pi}\right]\left\{\left[\frac{Q}{\lambda}\sin\frac{l\pi}{h}(z-h)\int_0^t\overline{\chi_1}(m,n,t')\exp[-k\eta^2(t-t')]\,dt'\right]-\left[\sin\frac{l\pi}{h}z\int_0^t\overline{\chi_2}(m,n,t')\exp[-k\eta^2(t-t')]\,dt'\right]\right\}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{a} \tag{23}$$

$$\sigma_{yy} = \frac{-8\alpha k\pi E}{h^2a^2}\sum_{l,m,n=1}^{\infty}l(-1)^{-l}\left[\frac{\eta^2-\frac{n^2\pi^2}{a^2}}{\eta^2\cos l\pi}\right]\left\{\left[\frac{Q}{\lambda}\sin\frac{l\pi}{h}(z-h)\int_0^t\overline{\chi_1}(m,n,t')\exp[-k\eta^2(t-t')]\,dt'\right]-\left[\sin\frac{l\pi}{h}z\int_0^t\overline{\chi_2}(m,n,t')\exp[-k\eta^2(t-t')]\,dt'\right]\right\}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{a} \tag{24}$$

$$\sigma_{zz} = \frac{-8\alpha k\pi E}{h^2a^2}\sum_{l,m,n=1}^{\infty}l(-1)^{-l}\left[\frac{\eta^2-\frac{l^2\pi^2}{a^2}}{\cos l\pi}\right]\left\{\left[\frac{Q}{\lambda}\sin\frac{l\pi}{h}(z-h)\int_0^t\overline{\chi_1}(m,n,t')\exp[-k\eta^2(t-t')]\,dt'\right]-\left[\sin\frac{l\pi}{h}z\int_0^t\overline{\chi_2}(m,n,t')\exp[-k\eta^2(t-t')]\,dt'\right]\right\}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{a} \tag{25}$$

SPECIAL CASE AND NUMERICAL RESULTS AND DISCUSSION:

Setting $\chi_1(x,\ y,t) = (1-e^t)x(a-x)y^2(3a-2y)$

$\chi_2(x,\ y,t) = (1-e^t)x(a-x)y^2(3a-2y)e^h$

In the equation (17) and(18) ,one obtains

$$T(x,\ y,\ z,\ t) = \frac{8k\pi}{h^2a^2}\sum_{l,m,n=1}^{\infty}l(-1)^{-l}\left[\frac{4(3a-2)a^6}{m^3n^2\pi^4}\right][(-1)^n+(-1)^{m+n}]\left[\frac{k\eta^2(1+e^t)-\exp[-k\eta^2t]+1}{k\eta^2[k\eta^2+1]}\right]\left[\frac{Q}{\lambda}\sin\frac{l\pi}{h}(z-h)-e^h\sin\left(\frac{l\pi}{h}\right)z\right]\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{a} \tag{26}$$

$$\chi_3(x,\ y,\ \ t) = \frac{8k\pi}{h^2a^2}\sum_{l,m,n=1}^{\infty}l(-1)^{-l}\left[\frac{4(3a-2)a^6}{m^3n^2\pi^4}\right][(-1)^n+(-1)^{m+n}]\left[\frac{k\eta^2(1+e^t)-\exp[-k\eta^2t]+1}{k\eta^2[k\eta^2+1]}\right]\left[\sin\frac{l\pi}{h}(\xi-h)-e^h\sin\left(\frac{l\pi}{h}\right)\xi\right]\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{a} \tag{27}$$

Dimensions of the square plate
Length, breadth of square plate a = 2cm
The thickness of the square plate, h=2 cm
$\xi = 0.5cm$
To interpret the numerical computations, we consider the material properties of a copper (pure) square plate with the material properties.
Poisson ratio, $\upsilon = 0.35$
Thermal expansion coefficient, α(cm/cm-^{0}C) = 16.5×10^{-6}
Thermal diffusivity, κ (cm^2/sec) = 112.34×10^{-6}
Youngs Modulus. E=120GPa

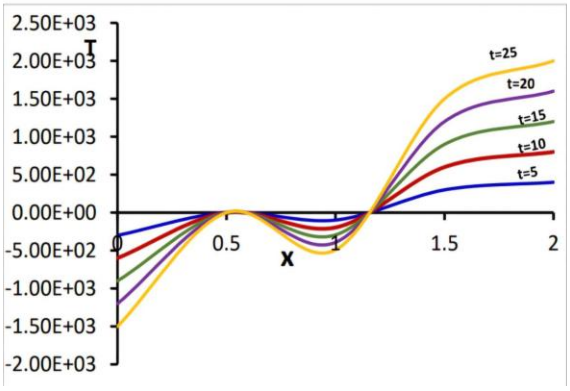

Figure 1: Graph of T versus x for different values t

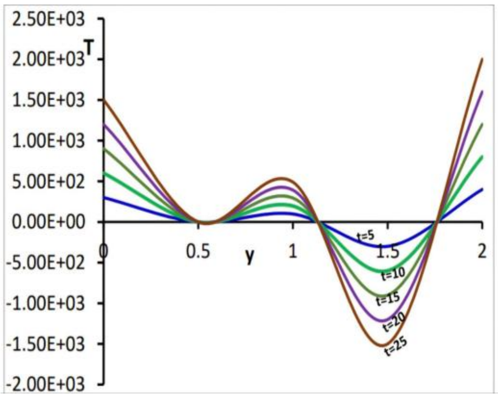

Figure 2: Graph of T versus y for different values t

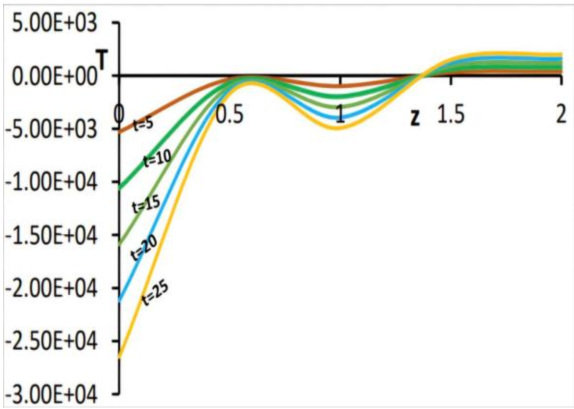

Figure 3: Graph of T versus z for different values t

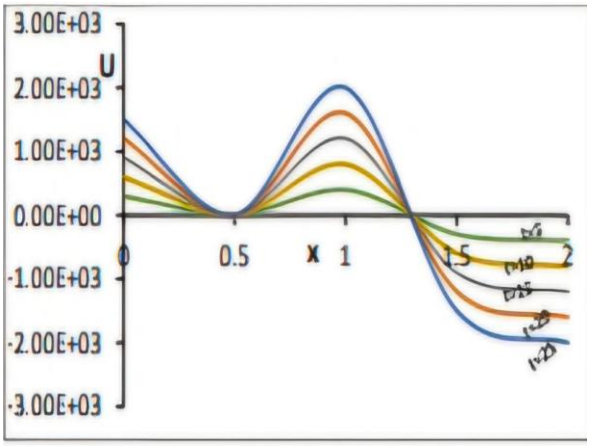

Figure 4: Graph of U versus x for different values t

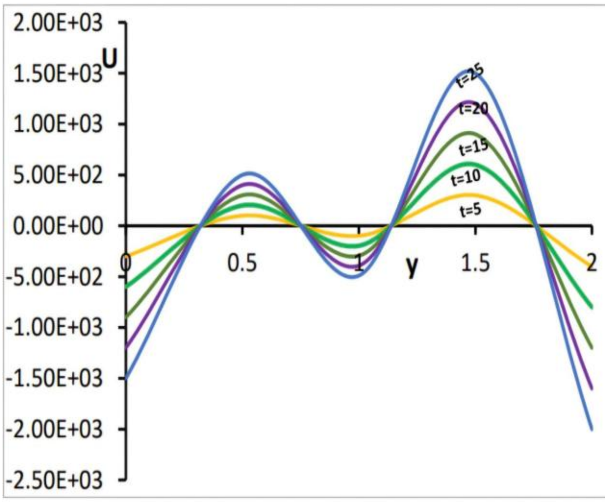

Figure 5 : Graph of U versus y for different values t

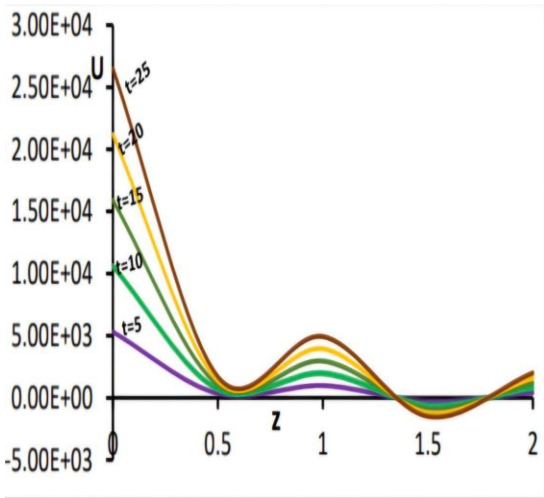

Figure 6: Graph of U versus z for different values t

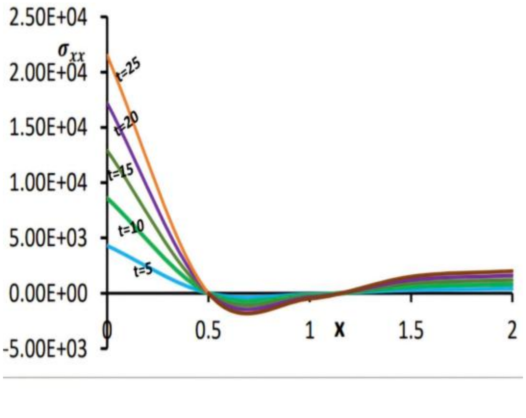

Figure 7: Graph of σ_{xx} versus x for different values t

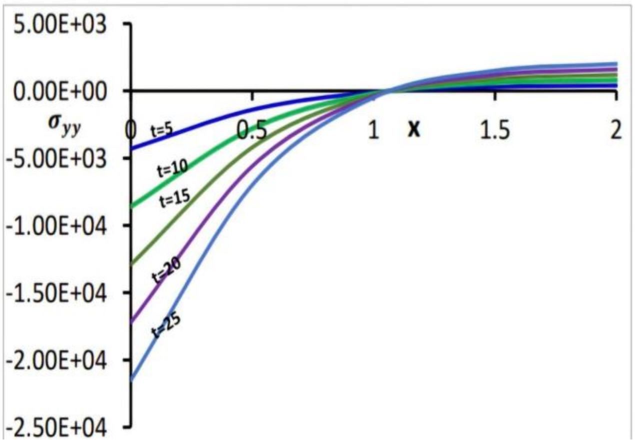

Figure 8: Graph of σ_{yy} versus x for different values t

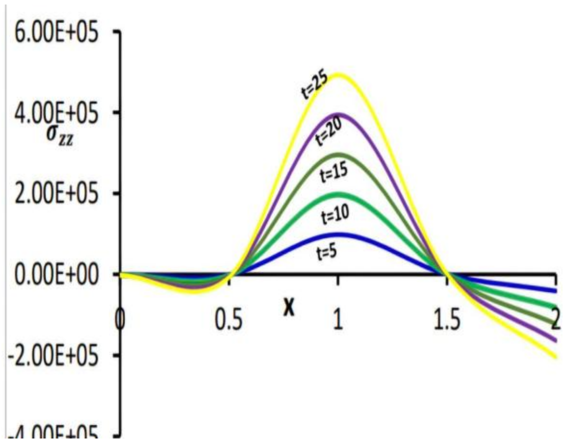

Figure 9: Graph of σ_{zz} versus x for different values t

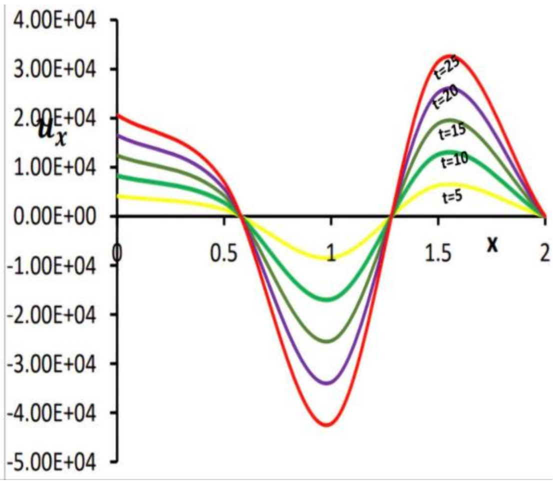

Figure10: Graph of u_x versus x for different values t

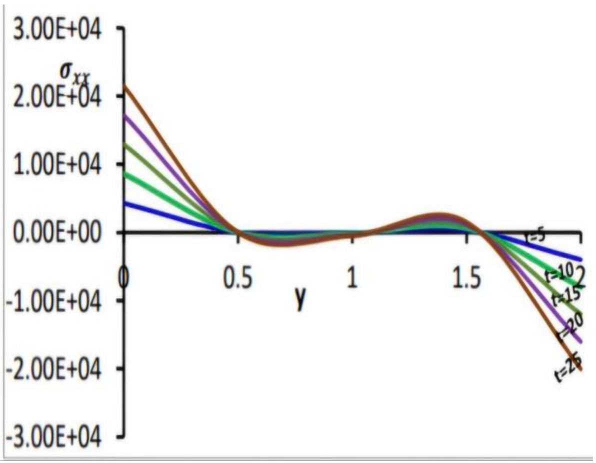

Figure 11: Graph of σ_{xx} versus y for different values t

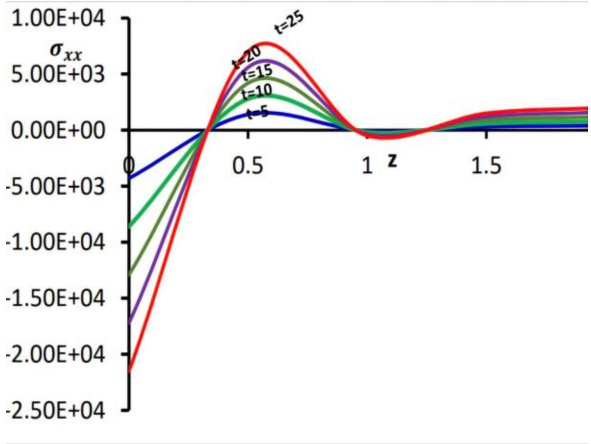

Figure 12: Graph of σ_{xx} versus z for different values t

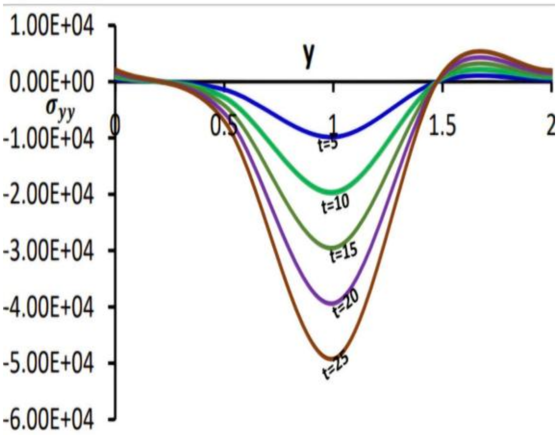

Figure 13: Graph of σ_{yy} versus y for different values t

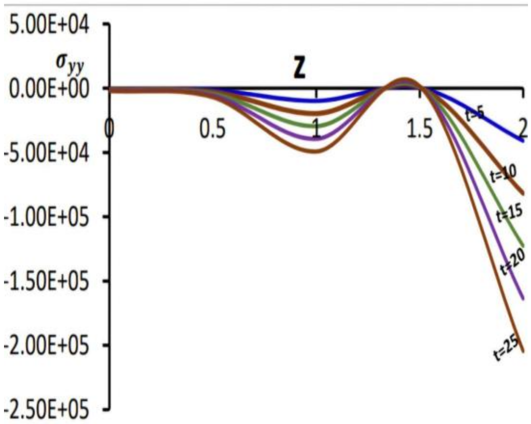

Figure 14: Graph of σ_{yy} versus y for different values t

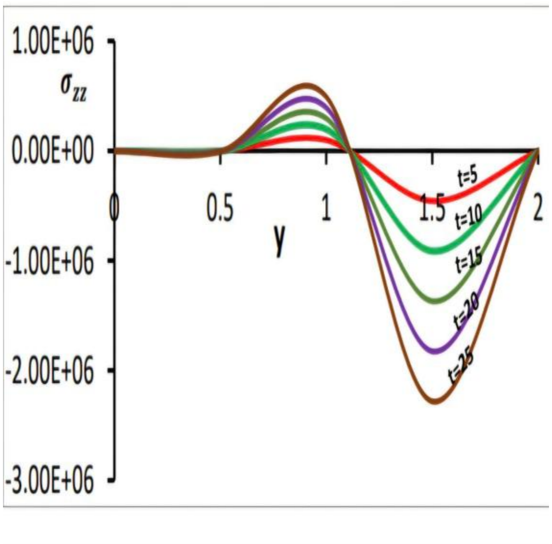

Figure 15: Graph of σ_{zz} versus y for different values t

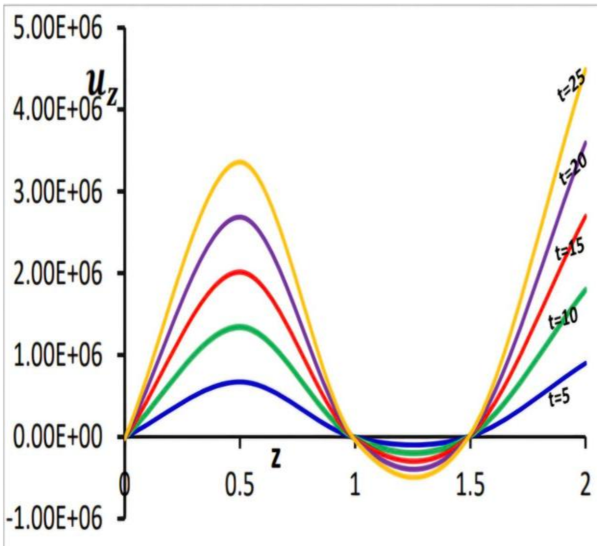

Figure 16: Graph of u_z versus z for different values t

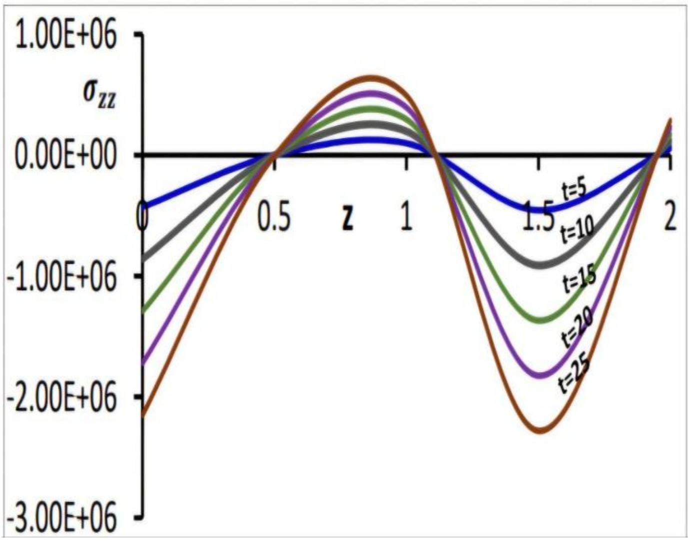

Figure 15: Graph of σ_{zz} versus z for different values t

CONCLUSION :

In this study one treated analytical study of the quasi-static thermoelastic problem of the square plate occupying region space$D$: $0 \leq x \leq a,\ 0 \leq y \leq a,\ 0 \leq z \leq h$. Under given initial and boundary conditions solution of the problem have been determined with the help of Integral transform and Laplace transform techniques . Numerical calculations are carried out for the particular case of the Square Plate made up of copper (pure) metal by assigning suitable values to the parameters and functions in the equations and results are explained graphically.

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