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SOLUTION FOR EINSTEIN – CARTAN DUST SPHERE

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Abstract:

The present paper provides solutions of E-C field equations for static dust sphere with spin by choosing a suitable form of effective density as $\overline{\rho} = \rho_0 \left(1 - \frac{r^2}{r_0^2}\right)$ we have also found various physical parameters like pressure matter density and spin density. Further we have fixed the constant using boundary conditions.

Key words: Dust sphere, effective density, pressure, spin, coframe.

1. INTRODUCTION: Prasanna [18] in (1975) found the analogs of some solution of fluid sphere of Tolman in E – C theory [14,15]. In his solution the equations corresponding to the classical condition of hydrostatic equilibrium was rather artificially split up into two separate equations so that a so – called "spin conservation equation" is satisfied. In effect this meant that spin does not provide any force (in the classical sense) of the equilibrium. But this assumption led to the difficulty that the derivative of the metric tensor components could not be made continuous on the boundary, which besides being a violations of Licknerowicz in (1955), seems difficult to accept. The junctions conditions were later discussed at some length by Kuchowicz in [11,12]. According to the junction conditions of the last mentioned workers the pressure on the surface of a Weyssonhoff fluid (Weyssonhoff and Ruabe [26]) sphere does not necessarily vanish. They, however, reached the erroneous conclusion that such a sphere doed not bounce. The error was later flashed in 1976 [9]. Raychaudhari [20(a)] has discussed the boundary conditions in the E – C theory for non – static fluid spheres. He has exhibited a particular solution for a dust sphere bouncing from a minimum volume.

Hehl, Heyde and Kerlick [9] have considered the Einstein's the field equations with Spin and Torsion U_4 theory to describe correctly the gravitational properties of matter on a microphysical level. They have shown how the singularities theorems of Penrose [17] and Hawking [6] must be modified to apply in E-C theory. Prasanna [18] has solved Einstein – Cartan field equations for a perfect fluid distribution and adopting Hehl's [7,8] approach, and Tolmans technique [25], obtained a number of solutions . Arkuszewski et al. [2] described the junction conditions in Einstein- Cartan theory. Raychaudhari and Banerji [20] considered collapsing spheres in Einstein- Cartan theory and showed that it bounces at a radius greater than the Schwarzschild radius. Banerji [3] has pointed out that Einstein – Cartan sphere must bounce outside the Schwarzschid radius if it bounces at all.

Singh and Yadav [22] studied the static fluid spheres in E – C theory and obtained a solution in an analytic from by the method of quadrature. Som and Bedran [24] got the class of solutions that represent a static incoherent spherical dust distribution in equilibrium under the influence of spin. Other workers in this line are Krorie et. al. [10], Mehra and Gokhroo [13], Suh {23], Maurya and Gupta [15], Yadav et al [27, 28], Amorim [1], Chatterji [4], Purushottam and Yadav [19], Sah and Chandra [21] and Murad [16].

In this paper we have obtained solution of the Einstein – Cartan field equations for static dust sphere with non – zero spin density taking a suitable choice of effective density. The constants have been fixed by using boundary conditions.

2. THE FIELD EQUATIONS:

We take Einstein – Cartan field equations in the form given by

(2.1) R^i_j = - $\frac{1}{2}$ R δ^i_j = -Xt^i_j

(2.2) Q^i_{jk}- δ^i_j Q^i_{jk} - δ^i_k Q^i_{ji} = Xs^i_{jk}

where Q^i_{jk} is torsion tensor, t^i_j is the canonical asymmetric energy momentum tensor, s^i_{jk} is the spin tensor and X = 8π.

We consider matter distribution given by the spherically symmetric metric

(2.3) dS^2 = e^v dt^2 - e^λdr^2 - r^2 $(d\theta^2 + \ sin^2\theta \ d\phi^2)$

where λ and γ are functions of r, we use comoving coordinates with U_4 velocity U^i = δ^i_4. The orthonormal coframe is chosen as

(2.4) θ^i = $e^{\lambda/2}$ dr, θ^2 = rdθ,

θ^3 = r Sinθ dϕ,

θ^4 = $e^{v/2}$ dt

So that,

g_{ij} = diag (1, -1, -1, -1).

When we assume a classical description of spin, we have

(2.5) $S_{ij}^{k} = S_{ij}u^{k}$

with

$S_{ij}u^{j}$ = 0.

where S_{ij} is the antisymmetric tensor of spin density. In the case of spherically symmetry, the tensor S_{ij} has the only non vanishing independent component S_{23} = K (say) and the non – zero parts of s_{jk}^{i} are

(2.6) s_{23}^{4} = - s_{32}^{4} = K

Hence from Einstein – Cartan equation (6.2.2), the non zero components of Q_{jk}^{i} are

(2.7) Q_{23}^{4} = - Q_{32}^{4} = -XK

Thus for a perfect fluid distribution with isotropic pressure p and matter density ρ the field equations (6.2.1) finally reduce to (Prassanna) [18]).

(2.8) $8\pi p = 16\pi^2 K^2 - \frac{1}{r^2} + e^{-\lambda}\left(\frac{1}{r^2} + \frac{v'}{r}\right)$

(2.9) $8\pi\rho = 16\pi^2 K^2 - \frac{1}{r^2} - e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right)$

(2.10) $\frac{e^{\lambda}}{r^2} = \frac{1}{r^2} - \frac{v'^2}{4} - \frac{v''}{2} + \frac{v'\lambda'}{4} + \frac{v'+\lambda'}{2r}$

where dashes stand for derivative w.r.t. r. The conservation laws gives us the relation

(2.11) $\left[p' + \frac{1}{2}(\rho + p)v'\right] + K\left(K' + \frac{1}{2}Kv'\right) = 0$

If we use the equilibrium condition given by

(2.12) $p' + \frac{1}{2}(\rho + p)\, v' = 0$

We get

(2.13) $K' + \frac{1}{2}Kv' = 0$

From (6.2.13) we have

(2.14) $K = He^{-v/2}$,

where H is a integration constant.

Following Hehl [7, 8], if we define effective pressure $\overline{p}$ and effective density $\overline{\rho}$ as

(2.15) $\overline{\rho} = \rho - 2\pi K^2$,

$\overline{p}$ = p -.....- 2 πK^2,

Thus the equations (2.8) and (2.9) finally reduce to

(2.16) $8\pi\overline{p} = -\frac{1}{r^2} + e^{-\lambda}\left(\frac{1}{r^2} + \frac{v'}{r}\right)$,

(2.17) $8\pi\overline{\rho} = \frac{1}{r^2} + e^{-\lambda}\left(-\frac{1}{r^2} + \frac{\lambda'}{r}\right)$,

Equation (2.10) remains as such.

The equation (2.11) now be comes

(2.18) $\frac{d\overline{p}}{dr} + \frac{1}{2}(\overline{\rho} + \overline{p})v' = 0$

We use the boundary conditions

(2.19) $[e^{-\lambda}]_{r=r_0} = [e^{v}]_{r=r_0} = \left(1 - \frac{2m}{r_0}\right)$,

$\overline{p} = 0$ at $r = r_0$ (radius of fluid sphere)

The mass m of fluid sphere is given by

(2.20) $m = 4\pi\int_o^{r_0} \overline{\rho}\, r^2\, dr = 4\pi\int_0^{r_0} P\, r^2\, dr - 8\pi^2$

$\int_0^{r_0} K^2(r)\, r^2$ dr.

Thus the correction in mass is

$8\pi^2 \int_0^{r_0} K^2(r)\, r^2$ dr.

3. SOLUTION OF THE FIELD EQUATIONS:

The equations (2.10) – (2.18) describe a system of classical spin with matter density ρ having pressure p zero. Some and Bedran [24] have assumed

(3.1) $\overline{\rho} = \rho_0 \frac{r^2}{r_b^2}$

and obtained a solution along with Schwarzchild exterior solution given by

(3.2) $dS^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2\, d\theta^2 - r^2\, sin^2\theta\ \varphi\ d\,\phi^2$

The effective density distribution given by (3.1) is vanishing at the centre r = 0 and it goes on increasing along the radius. Thus the fluid distribution chosen by Som and Bedran [24] is not regarded as physical. To make the distribution physically acceptable we take the effective density $\overline{\rho}$ as

(3.3) $\overline{\rho} = \overline{\rho}_0 \left(1 - \frac{r^2}{r_b^2}\right)$

where $\overline{\rho}_0$ is the effective density at the centre. Then from the field equations (2.16) – (2.18), we have

(3.4) $e^{-\lambda} = 1 - \frac{8\pi\rho_0}{15}\left(5r^2 - \frac{3r^4}{r_b^2}\right) + \frac{\alpha}{r}$

Where α is an integration constant. In order to avoid singularity at the origin α should be zero. Therefore taking α =0, equation (3.4) reduces to

(3.5) $e^{-\lambda} = 1 - \frac{8\pi\rho_0}{15}\left(5r^2 - \frac{3r^4}{r_b^2}\right)$

while v is given by

(3.6) $e^{v} = \left[A cos\left(\frac{\Psi}{2}\right) + B Sin\left(\frac{\Psi}{2}\right)\right]^2$

4. Some physical features:

Pressure, matter density and spin density for the distribution are found to be

(4.1) $8\pi p = 16\pi^2 H^2 \left[A\ Cos\left(\frac{\Psi}{2}\right) + B Sin\left(\frac{\Psi}{2}\right)\right]^{-2}$

(4.2) $8\ \pi\rho = 16\pi^2 H^2 \left[A\ Cos\left(\frac{\Psi}{2}\right) + B Sin\left(\frac{\Psi}{2}\right)\right]^{-2} + 8\pi p_0\left(1 - \frac{r^2}{r_b^2}\right)$

(4.3) $K = H\left[A\ Cos\left(\frac{\Psi}{2}\right) + B Sin\left(\frac{\Psi}{2}\right)\right]^{-1}$

with A and B being constants and

(4.4) $\Psi = \log\left[\frac{r^2}{r_b^2} - \frac{5}{6} + \left\{\left(\frac{r_2}{r_b^2}\right)^2 - \frac{5}{3}\frac{r^2}{r_b^2} + \frac{5}{8\pi r_b^2 p_o}\right\}^{1/2}\right]$

Using boundary condition as in (6.2) the constants A, B and H are found to be

(4.5) $H = \left\{\frac{p_{rb}}{2Љ}\right\}^{1/2}\left[ACos\left(\frac{\Psi_1}{2}\right) + Bsin\left(\frac{\Psi_1}{2}\right)\right]$

(4.6) $A = \left(1 - \frac{16\pi r_b^2 p_0}{15}\right)^{1/2} Cos\left(\frac{\Psi_1}{2}\right) - \frac{2r_b}{3}\left(\frac{2\pi p_0}{5}\right) Sin\left(\frac{\Psi_1}{2}\right)$

(4.7) $B = \left(1 - \frac{16\pi r_b^2 p_0}{15}\right)^{-1/2} Sin\left(\frac{\Psi_1}{2}\right) - \frac{2r_b}{3}\left(\frac{2\pi p_0}{5}\right) Cos\left(\frac{\Psi_1}{2}\right)$

5. Conclusion and further scope:

Our solutions given in this chapter are physically realistic. The effective density is not vanishing at the centre r = 0 and it goes on decreasing along the radius where as in solution given by Som and Bedran[24] effective density is zero at the centre and increases along radius which is not regarded as physical. Further study on this topic can be made using some different condition on effective density but the solution should be physical and avoided singularity.

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