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OROGRAPHIC EFFECT OF THE ASSAM-BURMA HILLS IN INDIA ON A BAROTROPIC AIR-STREAM

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ABSTRACT

An attempt has been made to obtain an analytical solution for 3-D meso-scale lee wave across the Assam-Burma hills (ABH) in India for an idealized barotropic mean flow. ABH has been approximated by two 3-D elliptical barriers, separated by a valley of some finite distance and is broadly north-south (NS) oriented. For more simplicity, the basic flow has been assumed to have two components U and V, normal to the major ridges of the elliptical barriers and parallel to the major ridges of the elliptic barriers respectively and a rectangular co-ordinate system in which, x-axis points towards east, y-axis points towards north and z-axis vertically upwards are considered and also U,V and Brunt-Vaisala frequency (N) are assumed to be invariant with vertical. The perturbation vertical velocity (w') and stream line displacement (η') are expressed as a double integral. These two integrals have been evaluated asymptotically. Result of the study may be briefly summarized as followings:

(i) The contours of the perturbation vertical velocity (w') and stream line displacement (η') in the asymptotic solution are approximately crescent shaped in the horizontal plane and spread laterally with vertical.

(ii) w' and η' in asymptotic solution decay downwind of the barrier along the line $Uy - Vx = 0$.

Key words: Assam-Burma hills, Asymptotic solution, 3D lee wave.

1. Introduction

It is known that the airflow over mountains or hilly terrain is more disturbed than over the flat country. Stably stratified airflow across a meso scale mountain gives rise to Internal gravity wave, known as mountain wave, which is a potential aviation hazard. Theoretical studies of the perturbation in a stably stratified air stream by an obstacle may broadly be divided into two categories. In one category the obstacle is assumed to have an infinite extension in the crosswind direction, so that the flow essentially becomes two-dimensional (2-D). In the other category the obstacle is assumed to have finite extension in the crosswind direction and the flow becomes essentially three-dimensional (3-D). The study on 2-D mountain wave problem was first addressed by Lyra (1943). He obtained lee wave solution using Green's functions, which decreased downstream and increased upward. Besides, studies on 2-D mountain wave problem may be found in Queney (1947,1948), Scorer (1953, 1954, 1956) etc.

Scorer and Wilkinson (1956) first studied three dimensional lee waves of non uniform air stream over an isolated hill. In their result, lee waves were confined within a wedge-shaped region, the corner of which being vertical and through the hill top, where the half angle of the wedge was dependent on the air stream character.

Wurtele (1957) considered the 3-D lee waves of incoming wind (U) and buoyancy frequency (V) to be

independent of height over the orographic barrier in the form of semi-infinite plateau of height 'h' with

narrow width '2d' in the crosswind direction. He predicted the region of updraft, which had 'horseshoes' shaped. Crapper (1959) presented a 3-D small perturbation approach of waves produced in a stably stratified air stream flowing over a mountain. He obtained the fundamental solution for a doublet disturbance in an air stream in which Scorers parameter remains constant and then it was extended to that for a disturbance caused by a circular mountain in the same air stream. He showed that circular mountain can give rise to waves which have greater amplitude than those produced by an infinite ridge in the same air stream.

Sawyar (1962) studied three dimensional mountain waves problem for a vertically variation of the amplitude of the standing waves when the wind varied with height. He showed the solutions for specified two or three layer atmosphere to determined possible wavelengths in the horizontal direction for lee waves. Also he obtained approximately 'crescent ' shaped updraft region with concave downwind.

Das (1964) studied the influence of Himalaya, approximated as a layer 3-D circular mountain, using a linear baroclinic model which included the variation of Coriolis parameter (*f*) with latitude. He considered the effect of coriolis force and shown that the nodal lines in his solution were system of concentric circle. Krishnamurty (1964) studied the effect of realistic, steady and non-linear mountain waves problem. He showed that non-linear effects tend to move the disturbance upward at the upstream side of the mountain but does not cause changes in the flow pattern at the downstream side.

Foldvic and Wurtele (1967) have studied numerically the transient nature of lee waves for idealized and realistic air-stream at various time intervals. They have showed the intensity of down-slope wind is intensified and that up-slope winds weakened, thus producing hydraulic jumps over the lee slope in some areas.

Onishi (1969) has solved three dimensional mountain waves problem for arbitrary upstream condition.

He obtained 3-D linearized equations by including friction in the governing equations. Pekelis (1971) developed his 2-D model to solve linearized 3-D problem. He obtained Vertical velocity fields and compared well with Sawyar (1962).

De (1973) showed that air stream characteristics across Assam-Burma hills during winter season are favorable for the occurrence of orographic gravity waves. Using satellite picture, he also documented the observational evidence of orographic gravity wave across the Assam-Burma hills.

Smith (1978) presented mountain wave problem over the Blue-ridge mountain in the central Appalachians. He determined the pressure drag during the first two week of January 1974, several periods with significant wave drag were observed by him with pressure differences typically 50N/m^2 across the ridge. Smith (1979) considered three dimensional lee wave in a stably stratified airflow consists of transverse lee wave and divergent lee wave. Smith (1980) studied the stratified hydrostatic lee waves over a bell shaped 3-D isolated circular mountain. He obtained the solutions for various part of the flow by using quasi-numerically technique where zonal wind (U) and buoyancy frequency (V) are constants with height. Somieski (1981) considered mountain waves problem for the stratified hydrostatic flow over 3-D circular mountain and including constant rotation and vertical wind shear of the mean flow. He solved the governing equations numerically. Sinha Ray (1988) presented a dynamical model for the perturbation vertical velocity over the orographic barrier. He included the friction and solved numerically by using perturbation technique.

Dutta (2001) addressed two dimensional frictionless mountain waves across western ghat, where basic flow (U) and Brunt- Vaisala frequency (N) are idealistic. He found out the momentum flux and energy flux associated with mountain waves using by perturbation technique. Dutta et al. (2002) considered stably stratified airflow over a three dimensional meso-scale orographic barrier with elliptical contour. He solved the governing equations asymptotically and numerically. Dutta (2005) has developed three dimensional meso-scale numerical model then he applied this model to Western Ghats of India using real time RS data of Santacruz.

Das et al (2013) developed three dimensional mountain waves problem over Assam-Burma hills (ABH) associated with idealistic basic flow. They obtained the asymptotic solutions using perturbation

approach and compared with two dimensional problem of earlier authors.

In India, the problems of lee waves across the Assam-Burma hills was first addressed by De(1970) and subsequently by De (1971), Farooqui and De (1974), Dutta and Naresh Kumar (2005) etc. Farooqui and De (1974) used a two dimensional model to calculate the flow over a small obstacle and large obstacle across the Assam-Burma hills. De (1970,1971) computed wavelength of lee waves over Assam-Burma hills using an approach, similar to Sarker (1966,1967). However, above all studies on mountain wave across Assam-Burma hills are 2-D. Das et al (2013) studies 3-D lee waves across the Assam-Burma hills for idealized basic flow, where, both stability and wind in basic flow of one component (U) remain invariant with height.

From the foregoing discussions it appears that in most of the studies on 3-D mountain wave problem, the basic flow is assumed to consist of only that component (U), which is normal to the major ridge of the mountain. Those studies did not consider other component of basic flow (V), which is parallel to the major ridge of the mountain. But in the real atmosphere at any level horizontal wind may have both components, viz., the component normal to the major ridge as well as the component parallel to the major ridge. So, it is necessary to investigate, at least qualitatively, the effect of 'V' component on the pattern of perturbation vertical velocity (w') and stream line displacement (η') associated with 3-D lee wave.

The objective of the present study is to develop a 3-D lee wave model across the Assam-Burma hills with a basic flow having both the components 'U' and 'V' and thereby to study the effect of 'V' component.

2. Data

For the present study we have selected the data of the only station on the upstream side is Guahati (26.19^0N **Latitude** and 91.73^0E **Longitude). The average of 0000UTC and 1200UTC RS/RW data of Guahati for those dates, which corresponds to the observed lee waves across ABH, as reported by De (1970, 1971) and** Farooqui and De (1974), has been obtained from Archive of India Meteorological Department, Pune.

3. Methodology

In the present study an adiabatic, steady state, non rotational, laminar, non viscous, frictionless and Boussinesq 3-D barotropic mean flow with idealistic vertical variation of wind and temperature across the Assam-Burma hills, have been considered. Present study is similar to the study of Dutta (2003) in most of the aspects, except the lower boundary condition. Similar to Dutta (2003), in the present study also it is assumed that the basic consists of two components U and V are normal and parallel to the major ridge of the elliptical barriers respectively and they are constant with vertical and the buoyancy frequency (N) is also assumed to be constant with vertical and a rectangular co-ordinate system in which, x-axis points towards east, y-axis points towards north and z-axis vertically upwards is considered. Using the technique followed by Dutta (2003) , we obtain the following vertical structure equations for perturbation vertical velocity (w') and for perturbation vertical streamline displacement (η'):

$$\frac{\partial^2 \hat{w}_1}{\partial z^2} + \left[\frac{N^2(k^2+l^2)}{(Uk+Vl)^2} - \frac{1}{2\rho_0}\frac{d^2\rho_0}{dz^2} + \frac{1}{4\rho_0^2}\left(\frac{d\rho_0}{dz}\right)^2 - (k^2+l^2) \right]\hat{w}_1 = 0 \qquad (1)$$

$$\frac{\partial^2 \hat{\eta}_1}{\partial z^2} + \left[\frac{N^2(k^2+l^2)}{(Uk+Vl)^2} - \frac{1}{2\rho_0}\frac{d^2\rho_0}{dz^2} + \frac{1}{4\rho_0^2}\left(\frac{d\rho_0}{dz}\right)^2 - (k^2+l^2) \right]\hat{\eta}_1 = 0 \qquad (2)$$

Where $\hat{w}_1, \hat{\eta}_1$ are double Fourier transforms of w_1', η_1' respectively and $N = \sqrt{\frac{g}{\theta_0}\frac{d\theta_0}{dz}}$ is the Brunt-Vaisala frequency.

$$w'(x,y,z) = \left(\frac{\rho_0(0)}{\rho_0(z)}\right)^{1/2} w_1'(x,y,z) \qquad (3)$$

$$\eta'(x,y,z) = \left(\frac{\rho_0(0)}{\rho_0(z)}\right)^{1/2} \eta_1'(x,y,z) \qquad (4)$$

Since in the equations (1) and (2) the terms $\left(-\frac{1}{2\rho_0}\frac{d^2\rho_0}{dz^2}\right)$ and $\frac{1}{4\rho_0{}^2}\left(\frac{d\rho_0}{dz}\right)^2$ are less, by at least one order of magnitude, than the other terms in the square bracket, the equation (1) and (2) reduced to

$$\frac{\partial^2\hat{w}_1}{\partial z^2}+(k^2+l^2)\left[\frac{N^2}{(Uk+Vl)^2}-1\right]\hat{w}_1=0 \qquad (5)$$

$$\frac{\partial^2\hat{\eta}_1}{\partial z^2}+(k^2+l^2)\left[\frac{N^2}{(Uk+Vl)^2}-1\right]\hat{\eta}_1=0 \qquad (6)$$

Equations (5) and (6) are solved subject to the following boundary conditions:

(i) At the lower boundary stream line pattern follow the contour of the mountain,

(ii) At the upper boundary radiative boundary condition is imposed i.e., mountain waves are allowed to propagate vertically.

Now using the upper boundary condition (ii), the general solution of equation (14) and (16) can be taken as

$$\hat{w}_1(k,l,z)=Ae^{imz} \qquad (7) \text{ and}$$

$$\hat{\eta}_1(k,l,z)=Be^{imz} \qquad (8)$$

where A, B are constants to be determined using lower boundary condition and m is given by, $m^2=\left[\frac{N^2}{(kU+lV)^2}-1\right](k^2+l^2)$. Clearly m may be recognized as the vertical wave number of the vertically propagating mountain waves.

Now at the lower boundary i.e., at the surface the airflow follows the contour of the mountain, the profile of which is given by

$$h(x,y)=\frac{h_1}{1+\frac{x^2}{a^2}+\frac{y^2}{b^2}}+\frac{h_2}{1+\frac{(x-d)^2}{a^2}+\frac{y^2}{b^2}} \qquad (9)$$

The profile of ABH barrier (9) is given by Figure 1. In the present study the values of a, d_1, h_1 and h_2 are same as those in De (1971) and $b=2.5a$ as in Dutta (2005), as in Das et al (2013). Therefore, we take $a=20km, b=2.5a, d_1=45km, h_1=0.9km$ and $h_2=0.7km$ .If $\hat{h}(k,l)$ be the double Fourier transformation of $h(x,y)$, then expression of $\hat{h}(k,l)$ given as

$$\hat{h}(k,l)=2\pi ab\left(h_1+h_2e^{-ikd_1}\right)K_0\left(\sqrt{a^2k^2+b^2l^2}\right) \qquad (10)$$

Where $K_0(\sqrt{a^2k^2+b^2l^2})$ is the Bessel function of second kind of order zero. Details derivation for the expression are available in Das et al. (2013). At the lower boundary we have

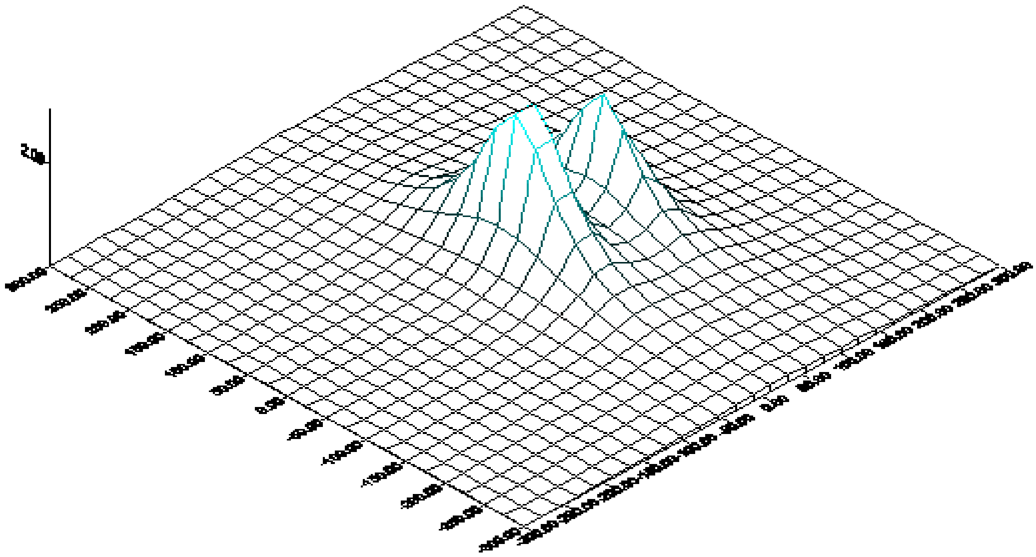


Figure 1. Profile of ABH barrier.

$$\eta'(x,y,0)=\eta_1'(x,y,0)=h(x,y).$$

$$\therefore \hat{\eta}(k,l,0)=\hat{\eta}_1(k,l,0)=\hat{h}(k,l)=2\pi ab\left(h_1+h_2e^{-ikd}\right)K_0(\sqrt{a^2k^2+b^2l^2})$$

Hence, $\mathrm{B}=2\pi ab\left(h_1+h_2e^{-ikd}\right)K_0(\sqrt{a^2k^2+b^2l^2})$.

Again the linearized lower boundary condition for w' may be given by

$$w'(x,y,0)=w'_1(x,y,0)=U\frac{\partial\eta'(x,y,0)}{\partial x}+V\frac{\partial\eta'(x,y,0)}{\partial y}$$

$$\therefore \hat{w}(k,l,0)=\hat{w}_1(k,l,0)=i(kU+lV)\hat{\eta}(k,l,0) \qquad (11)$$

Hence, $\mathrm{A}=2\pi iab\left(h_1+h_2e^{-ikd}\right)K_0(\sqrt{a^2k^2+b^2l^2})$.

Using the values of A and B in (7) and (8) respectively, we get

$$\hat{w}_1(k,l,z)=2\pi iab\left(h_1+h_2e^{-ikd}\right)K_0(\sqrt{a^2k^2+b^2l^2})e^{imz} \qquad (12)$$

$$\hat{\eta}_1(k,l,z)=2\pi ab\left(h_1+h_2e^{-ikd}\right)K_0(\sqrt{a^2k^2+b^2l^2})e^{imz} \qquad (13)$$

Therefore, the perturbation vertical velocity and stream line displacement at any point (x,y,z)

are given by respectively

$$w_1'(x,y,z) = \mathrm{Re}\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{w}_1(k,l,z)e^{i(kx+ly)}dkdl$$

$$= \frac{ab}{2\pi}\mathrm{Re}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} i(kU+lV)\left(h_1+h_2e^{-ikd}\right)K_0(\sqrt{a^2k^2+b^2l^2})e^{i(kx+ly+mz)}dkdl \quad (14)$$

And

$$\eta_1'(x,y,z) = \mathrm{Re}\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{\eta}_1(k,l,z)e^{i(kx+ly)}dkdl$$

$$= \frac{ab}{2\pi}\mathrm{Re}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(h_1+h_2e^{-ikd}\right)K_0(\sqrt{a^2k^2+b^2l^2})e^{i(kx+ly+mz)}dkdl \quad (15)$$

The above two equations (14) and (15) reduce to

$$w'(x,y,z) = \sqrt{\frac{\rho_0(0)}{\rho_0(z)}}w_1'(x,y,z) = c(z)\times R.P.of\,(I_1) \quad (16)$$

$$\eta'(x,y,z) = \sqrt{\frac{\rho_0(0)}{\rho_0(z)}}\eta_1'(x,y,z) = c(z)\times R.P.of\,(I_2) \quad (17)$$

Where $c(z) = \frac{ab}{2\pi}\sqrt{\frac{\rho_0(0)}{\rho_0(z)}}$ and

$$I_1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} i(kU+lV)\left(h_1+h_2e^{-ikd}\right)K_0(\sqrt{a^2k^2+b^2l^2})e^{i(kx+ly+mz)}dkdl \quad (18)$$

$$I_2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} i\left(h_1+h_2e^{-ikd}\right)K_0(\sqrt{a^2k^2+b^2l^2})e^{i(kx+ly+mz)}dkdl \quad (19)$$

Now, Dutta(2002) have shown that $\sqrt{\frac{\rho_0(0)}{\rho_0(z)}} \approx e^{\frac{(g-R^{\bullet}\gamma)z}{2R^{\bullet}\bar{T}}}$, hence, $c(z) = \frac{ab}{2\pi}e^{\frac{(g-R^{\bullet}\gamma)z}{2R^{\bullet}\bar{T}}}$.

Here also the integrals I_1 and I_2 are approximated by their asymptotic expansion using method of stationary phase. According to this method, first those points in the wave number (k,l) domain are found out, where the phase $(kx+ly+mz)$ is stationary. Those points are termed as saddle points. Then entire integrand is expanded in Taylor's series about the saddle point and the first term of the expansion is retained as the asymptotic approximation of the integrals, which is valid at far down wind location of the mountain. The asymptotic expansion for w' and η' is given by following :

$$w'\left(X_2,Y_2,Z_1\right)=-e^{\frac{\left(g-R^{\bullet}\gamma\right)z}{2R^{\bullet}\bar{T}}}\frac{\mathrm{A_1abN^3X_2Z_1}}{\mathrm{\rho_1R_1(U^2+V^2)}}\left[\mathrm{h_2}\sin\left(\frac{\mathrm{Z_1R_1}}{\rho_1}-\frac{\tau_0\cos\beta_0}{\sqrt{\mathrm{U^2+V^2}}}\mathrm{Nd}\right)+\mathrm{h_1}\sin\left(\frac{\mathrm{Z_1R_1}}{\rho_1}\right)\right]$$

(20)

$$\eta'\left(X_2,Y_2,Z_1\right)=e^{\frac{\left(g-R^{\bullet}\gamma\right)z}{2R^{\bullet}\bar{T}}}\frac{\mathrm{A_1abN^2}}{\mathrm{(U^2+V^2)}}\left[\mathrm{h_2}\cos\left(\frac{\mathrm{Z_1R_1}}{\rho_1}-\frac{\tau_0\cos\beta_0}{\sqrt{\mathrm{U^2+V^2}}}\mathrm{Nd}\right)+\mathrm{h_1}\cos\left(\frac{\mathrm{Z_1R_1}}{\rho_1}\right)\right]$$

(21)

where, $A_1=\dfrac{X_2Z_1\sqrt{\rho_1^{\,4}+\left(X_2Y_2\right)^2}}{2\rho_1^{\,3}R_1^{\,2}\sqrt{1+4\left(\dfrac{X_2Y_2Z_1R_1}{\rho_1^{\,4}+X_2^{\,2}Y_2^{\,2}}\right)^2}}K_0\left(Arg_0\right)$

$$Arg_0=\frac{\left(NX_2Z_1\right)\sqrt{(a^2(U\rho_1^{\,2}-VX_2Y_2)^2+b^2(V\rho_1^{\,2}+UX_2Y_2)^2)}}{\rho_1^{\,3}R_1\left(U^2+V^2\right)},$$

$$\rho_1^{\,2}=Y_2^{\,2}+Z_1^{\,2},\ R_1^{\,2}=X_2^{\,2}+\rho_1^{\,2},$$

$$X_2=\frac{N\left(Ux+Vy\right)}{\left(U^2+V^2\right)}\ ,\ Y_2=\frac{N\left(Uy-Vx\right)}{\left(U^2+V^2\right)}\ \text{and}\ Z_1=\frac{Nz}{\sqrt{U^2+V^2}}.$$

Details of the derivation of the equation (20) and (21) are given in **appendix I**.

4. Result and discussion

Now from (20) it is clearly that at $z=Z_1=0$ *i.e.*, at the ground surface the vertical velocity (w') vanishes except $x=0, y=0$. This represents the lee waves (Dutta, 2003), (Das et al, 2013). The geometrical description of wave pattern in 3-D is obtained by substituting $Y_2=0$, $\rho_1=Z_1, R_1^2=X_2^2+Z_1^2$, therefore $Arg_0=\dfrac{NX_2\sqrt{a^2U^2+b^2V^2}}{R_1\left(U^2+V^2\right)}$ and $A_1=\dfrac{X_2}{R_1^2}k_0(Arg_0)$. So at any level $(z\neq 0)$ for $Y_2=0$, w' and η' can be written as:

$$w'\left(X_2,0,Z_1\right)=$$

$$-e^{\frac{\left(g-R^{\bullet}\gamma\right)z}{2R^{\bullet}\bar{T}}}\frac{abN^3X_2^2K_0\left(\dfrac{NX_2\sqrt{a^2U^2+b^2V^2}}{\left(U^2+V^2\right)\sqrt{X_2^2+Z_1^2}}\right)}{2(X_2^2+Z_1^2)^{3/2}\left(U^2+V^2\right)}\left[\begin{array}{l}h_2\sin\left(\sqrt{X_2^2+Z_1^2}-\dfrac{\tau_0\cos\beta_0}{\sqrt{U^2+V^2}}Nd\right)+\\ h_1\sin\left(\sqrt{X_2^2+Z_1^2}\right)\end{array}\right] \qquad (22)$$

$\eta'\left(X_2,0,Z_1\right)=$

$$e^{\frac{\left(g-R^{\bullet}\gamma\right)z}{2R^{\bullet}\bar{T}}}\frac{abN^2X_2K_0\left(\frac{NX_2\sqrt{a^2U^2+b^2V^2}}{\left(U^2+V^2\right)\sqrt{X_2^2+Z_1^2}}\right)}{2(X_2^2+Z_1^2)\left(U^2+V^2\right)}\left[\begin{array}{l}h_2\cos\left(\sqrt{X_2^2+Z_1^2}-\frac{\tau_0\cos\beta_0}{\sqrt{U^2+V^2}}Nd\right)+\\ h_1\cos\left(\sqrt{X_2^2+Z_1^2}\right)\end{array}\right]$$

……(23)

From the above expressions of w' and η' are clearly seen that, both of them decay downstream of the elliptical barrier at a rate proportional to X_2^{-1} i.e., inversely proportional to the distance along the line

$Uy-Vx=0$. This may be attributed to the presence of Bessel function and the terms $\frac{X_2^2}{(X_2^2+Z_1^2)^{3/2}}$ and $\frac{X_2^2}{(X_2^2+Z_1^2)}$ respectively.

De (1973) investigated that the airstream characteristic across the ABH during winter season is favourable for the occurrence of the lee waves. Using the equations (20) and (21) the perturbation vertical velocity (w') and stream line displacement (η') are computed at different levels for a typical lee wave case cross the ABH during winter season, taken the order of magnitude of 'U' is 10m/sec. 'V' is of 7m/sec. and that of 'N' is 0.01/sec. Using the values of 'U', 'V' and 'V' we computed the values of w' and η' at four levels only, viz., 1.5km, 3km,6km and 9km above mean see level.

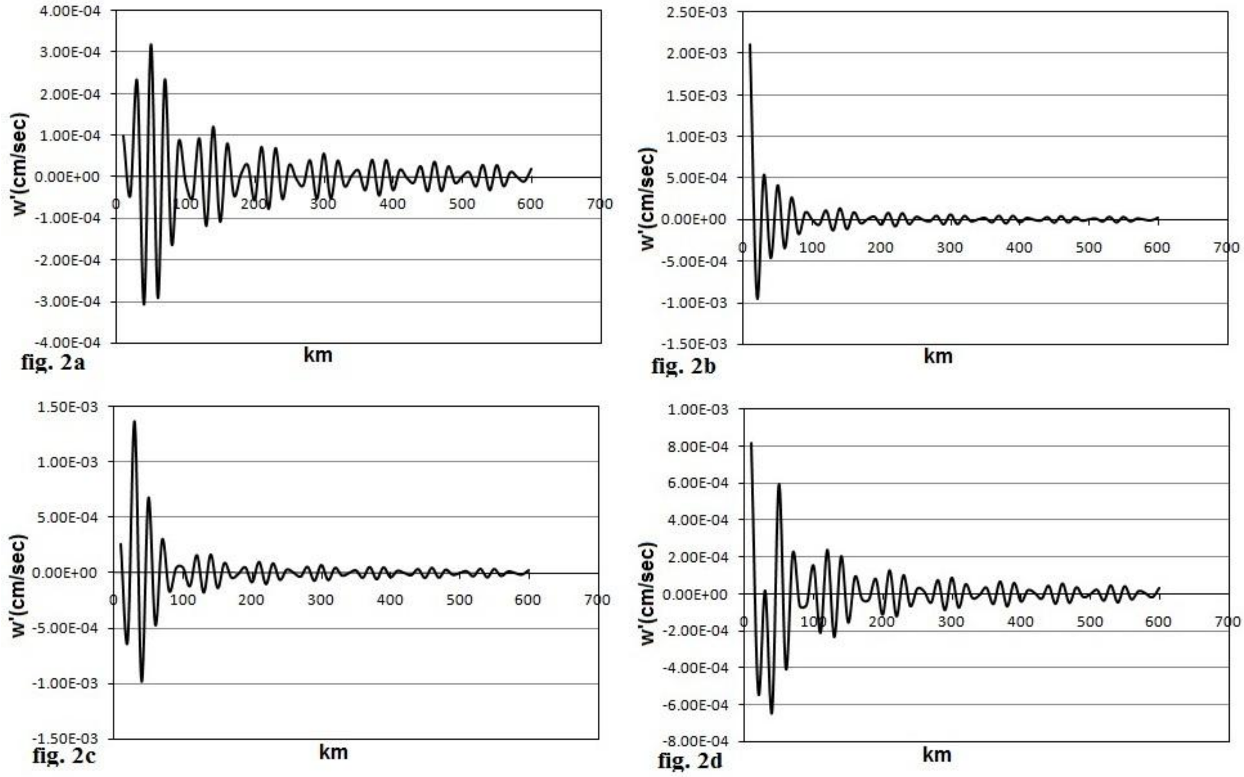


Figure 2(a-d). Down stream variation of $\acute{w}$ along the line $Uy - Vx = 0$, at 1.5km, 3km, 6km and 9km above mean see level respectively.

The downstream variation of the vertical velocity (w') and stream line displacement (η') have been shown by Figure 2(a-d) and Figure 3(a-d) respectively, along the line $Uy - Vx = 0$, at 1.5km, 3km, 6km and 9km above mean sea level, which approximately resemble to 850hPa, 700hPa, 500hPa and 300hPa respectively.

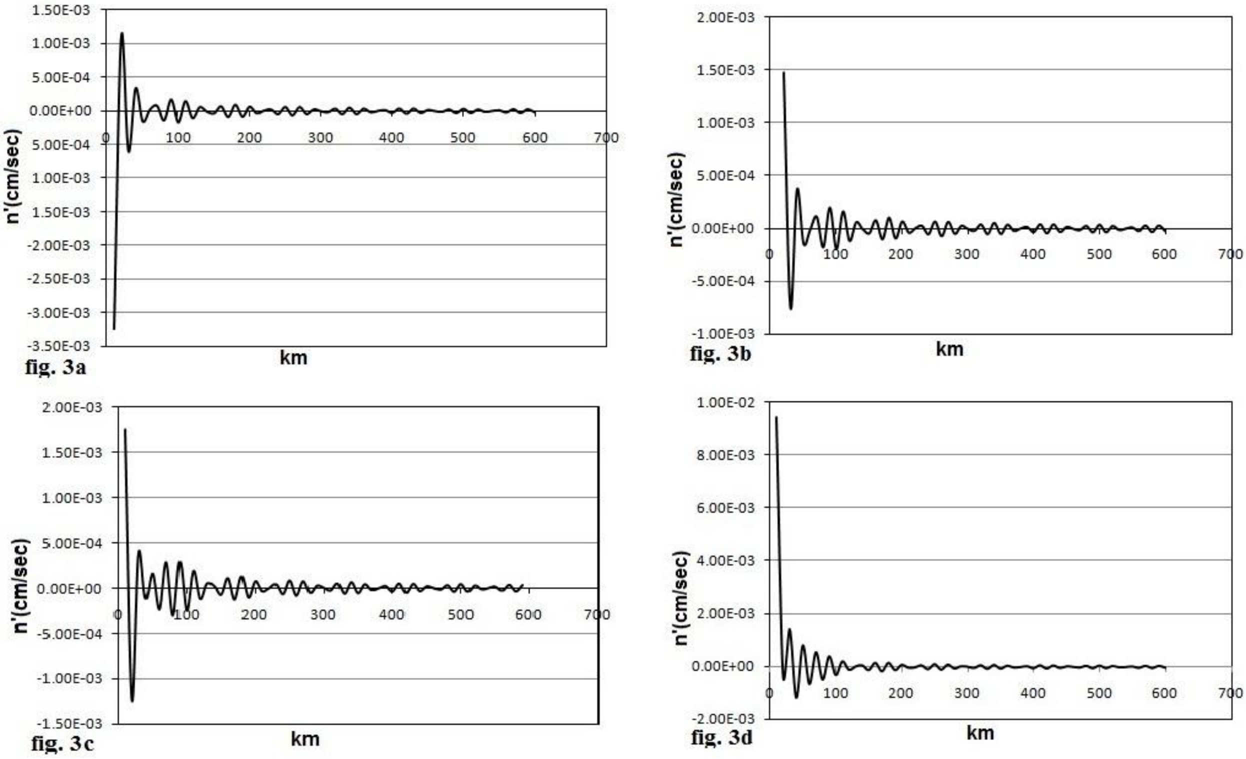
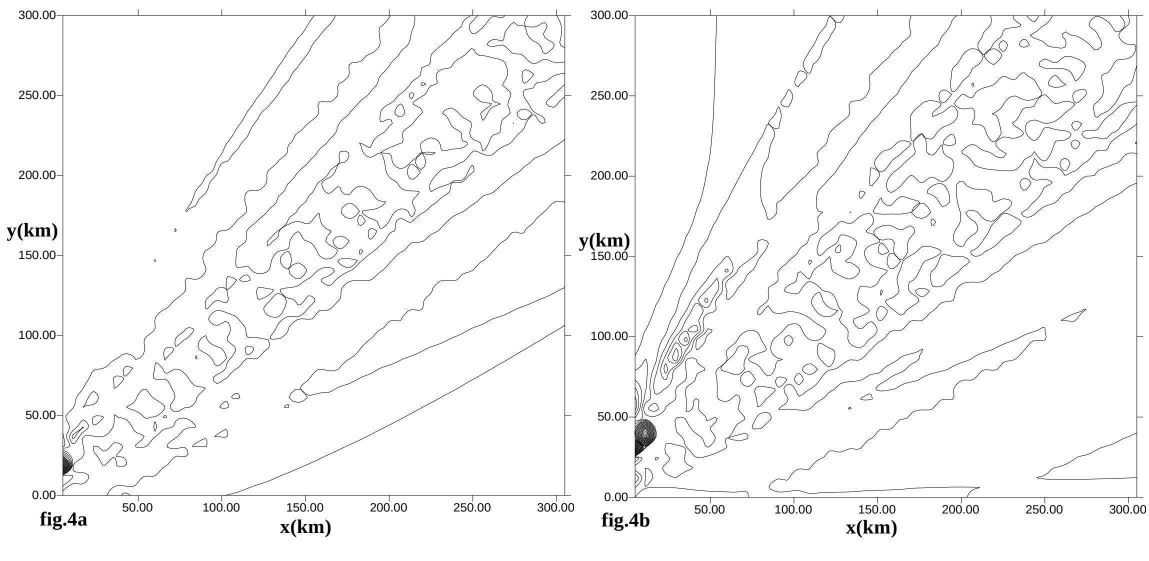


Figure 3(a-d). Down stream variation of $\acute{\eta}$ along the line $Uy - Vx = 0$, at 1.5km, 3km, 6km and 9km above mean see level respectively.

The figures show that both w' and η' are downstream decay in the amplitude of w' and η' respectively, along the line $Uy - Vx = 0$, in conformity with earlier investigators.

The contours of w' and η' have been shown in the Figure 4(a-d) and Figure 5(a-d) respectively, at 1.5km, 3km, 6km and 9km above mean sea level, which approximately resemble to 850hPa, 700hPa, 500hPa and 300hPa respectively.



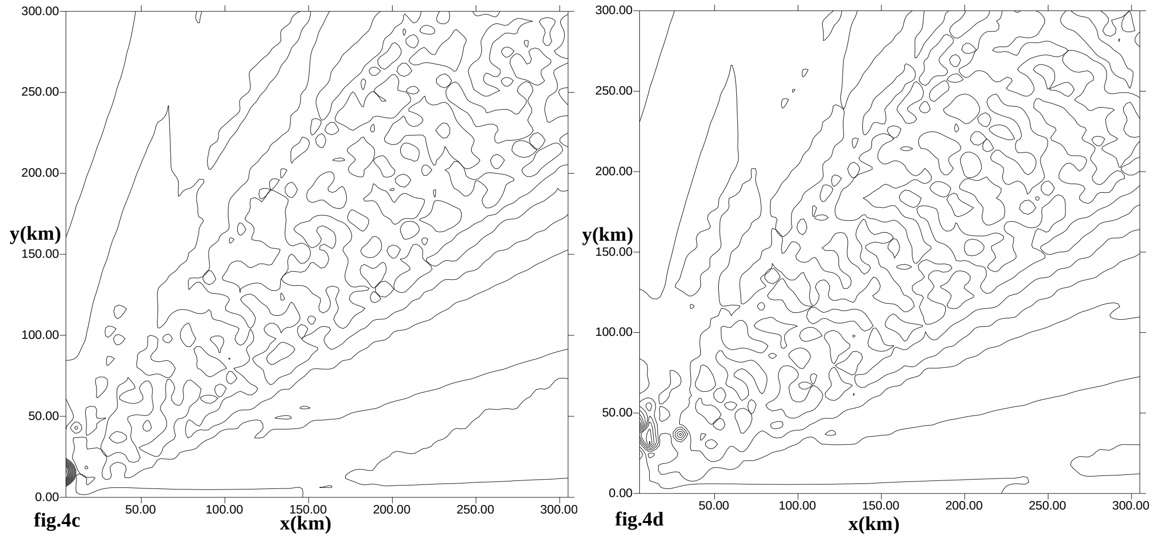
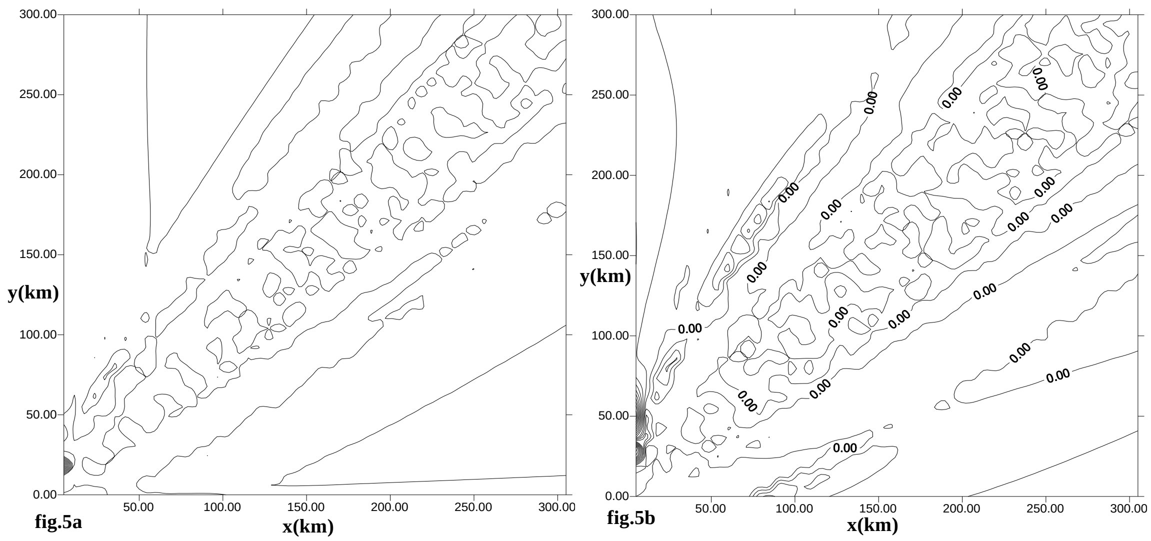


Figure 4(a-d). Contours of $\acute{w}$ at 1.5km, 3km, 6km and 9km above mean see level respectively.

The contours of both w' and η' are maximum updraft regions are approximately crescent shaped, tilt upwind and symmetric about the line $Uy - Vx = 0$, concave to the down wind direction and lateral spreading with vertical, implying divergent lee wave, which is in qualitative conformity with finding from earlier results. Wurtele(1957) obtained crescent shaped updraft region, symmetric about x-axis (i.e., about the line $y = 0$), taking constant basic flow with only U-component .



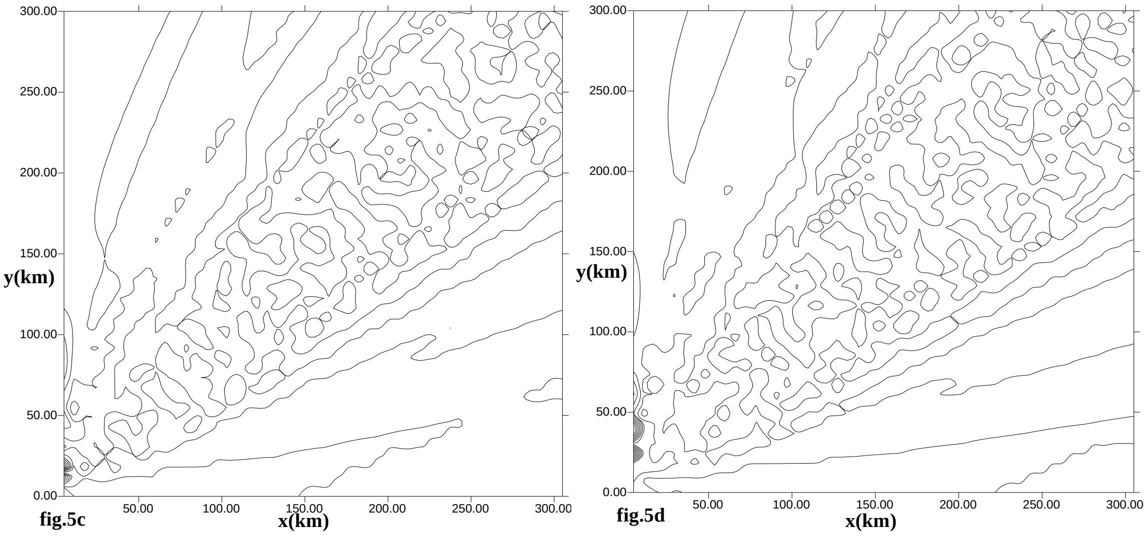


Figure 5(a-d). Contours of $\acute{\eta}$ at 1.5km, 3km, 6km and 9km above mean see level respectively.

Dutta (2003) has been shown that the maximum updraft regions are crescent shaped and symmetric about the line $Uy - Vx = const.$, taken constant basic flow of both components (U,V). Now due to the presence of V-component, there is a meridional forcing acting at all level and occurring the symmetric crescent shaped updraft region to rotate through an angle $tan^{-1}(V/U)$.

5. Conclusions

In this investigation, we have presented the effect of 3-D meso-scale lee wave across the Assam-Burma hills following an asymptotic approach. In the sequel, we have made some interesting observation. Moreover,

(i) The solutions for w' and η' show that along the line $Uy - Vx = 0$ both decay down wind of the barrier at a rate proportional to X_2^{-1} across the ABH.

(ii) In the horizontal plane the contours for w' and η' show crescent shaped updraft region, which are inclined at an angle $tan^{-1}(V/U)$.

(iii) Asymptotic solution for both w' and η' across the ABH, show upwind tilt along the line $Uy - Vx = 0$ and spread laterally with height .

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Appendix-I

Hsu's (1948) theorem: In the theorem, he has shown that if $\Phi(x,y), h(x.y)$ and $f(x,y)=e^{h(x,y)}$ are continuous functions defined on a region S such that,

(1) $\Phi(x,y),[f(x,y)]^n$ are absolutely integrable over S for n=0,1,2.............

(2) $\frac{\partial f}{\partial x},\frac{\partial^2 f}{\partial x^2},\frac{\partial f}{\partial y},\frac{\partial^2 f}{\partial y^2}$ exist and continuous over S.

(3) $h(x,y)$ has an absolute maximum value at an interior pt (x_0,y_0) such that

At (x_0,y_0) $\frac{\partial h}{\partial x}=\frac{\partial h}{\partial y}=0,\frac{\partial^2 h}{\partial x^2}\frac{\partial^2 h}{\partial y^2}-(\frac{\partial^2 h}{\partial x\partial y})^2>0$

(4) $\Phi(x,y)$ is continuous at (x_0,y_0) and $\Phi(x_0,y_0)\neq 0$.Now if C be an analytic curve passing through the point (x_0,y_0), such that the region S is divided into two sub regions S_1 and S_2. Then the integral

(5) $\iint\Phi(x,y)[f(x,y)]^n ds$ taken over either of S_1 and S_2 asymptotic to

$$\frac{\pi\Phi(x_0,y_0)[f(x_0,y_0)]^n}{n\sqrt{\left[\frac{\partial^2 h}{\partial x^2}\frac{\partial^2 h}{\partial y^2}-\left(\frac{\partial^2 h}{\partial x\partial y}\right)^2\right]_{(x_0,y_0)}}}$$

Now to evaluate the above integrals I_1 and I_2 in the equations (27) and (28) respectively, by substituting following :

$$x=\frac{X_1\sqrt{U^2+V^2}}{N},y=\frac{Y_1\sqrt{U^2+V^2}}{N},z=\frac{Z_1\sqrt{U^2+V^2}}{N} \text{ and } k=\frac{\lambda_1 N}{\sqrt{U^2+V^2}},l=\frac{\mu_1 N}{\sqrt{U^2+V^2}}$$

Now, $$m^2=\frac{N^2\left(\lambda_1^2+\mu_1^2\right)}{U^2+V^2}\left\{\frac{U^2+V^2}{\left(U\lambda_1+V\mu_1\right)^2}-1\right\}$$

And $$kx+ly+mz=\lambda_1 X_1+\mu_1 Y_1+Z_1\sqrt{\left\{\frac{U^2+V^2}{\left(U\lambda_1+V\mu_1\right)^2}-1\right\}}\sqrt{\left(\lambda_1^2+\mu_1^2\right)}$$

The above substitution, two integrals I_1 and I_2 reduce to

$$I_1=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} i\left(U\lambda_1+V\mu_1\right)\left(h_1+h_2e^{-\frac{\lambda_1 N}{\sqrt{U^2+V^2}}}\right)K_0\left(\frac{N\sqrt{a^2\lambda_1^2+b^2\mu_1^2}}{\sqrt{U^2+V^2}}\right)\frac{N^3}{\left(U^2+V^2\right)^{3/2}}e^{i\left(\lambda_1X_1+\mu_1Y_1+\gamma_1Z_1\right)}d\lambda_1d\mu_1$$

$$I_2=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(h_1+h_2e^{-\frac{\lambda_1 N}{\sqrt{U^2+V^2}}}\right)K_0\left(\frac{N\sqrt{a^2\lambda_1^2+b^2\mu_1^2}}{\sqrt{U^2+V^2}}\right)\frac{N^3}{\left(U^2+V^2\right)}e^{i\left(\lambda_1X_1+\mu_1Y_1+\gamma_1Z_1\right)}d\lambda_1d\mu_1$$

Where, $\gamma_1=\sqrt{\left\{\frac{U^2+V^2}{\left(U\lambda_1+V\mu_1\right)^2}-1\right\}}\sqrt{\left(\lambda_1^2+\mu_1^2\right)}$

Again following substitutions are made:

$X_1=r\cos\theta, Y_1=r\sin\theta$ and $\gamma_1=\tau\cos\beta, \mu_1=\tau\sin\beta$

Hence $\lambda_1X_1+\mu_1Y_1+Z_1\gamma_1=\tau r\cos\left(\beta-\theta\right)+Z_1\sqrt{\sec^2\left(\beta-\alpha\right)-\tau^2}=\sigma_1$ (say), where $\alpha=\tan^{-1}\left(\frac{V}{U}\right)$

Thus finally the integrals I_1 and I_2 reduce to

$$I_1=\int_{-\pi/2}^{\pi/2}\int_{0}^{\infty} i\tau\left(U\cos\beta+\gamma\sin\beta\right)\left(h_1+h_2e^{-i\frac{\tau\cos\beta}{\sqrt{U^2+V^2}}Nd}\right)K_0\left(\frac{N\tau\sqrt{a^2\cos^2\beta+b^2\sin^2\beta}}{\sqrt{U^2+V^2}}\right)\frac{N^3\tau e^{i\sigma_1}}{\left(U^2+V^2\right)^{3/2}}d\tau d\beta$$

$$I_2=\int_{-\pi/2}^{\pi/2}\int_{0}^{\infty}\left(h_1+h_2e^{-i\frac{\tau\cos\beta}{\sqrt{U^2+V^2}}Nd}\right)K_0\left(\frac{N\tau\sqrt{a^2\cos^2\beta+b^2\sin^2\beta}}{\sqrt{U^2+V^2}}\right)\frac{N^2\tau e^{i\sigma_1}}{\left(U^2+V^2\right)}d\tau d\beta$$

Now, to evaluate the above integrals I_1 and I_2, we follow the method of approximation of double integrals of large numbers **(Hsu, 1948)**.

We assume $\psi=\beta-\alpha$ and $\varphi=\theta-\alpha$ then $\sigma_1=\tau r\cos\left(\psi-\varphi\right)+Z_1\sqrt{\sec^2\psi-\tau^2}=h(\tau,\psi)$

Here $f\left(\tau,\psi\right)=e^{ih(\tau,\psi)}$ and $n=1$; for I_1, $\varphi\left(\tau,\psi\right)=p^3\tau^2K_0\left(\arg\right)\left\{U\cos\left(\psi+\alpha\right)+V\sin\left(\psi+\alpha\right)\right\}$ and for I_2, $\varphi\left(\tau,\psi\right)=p^2\tau K_0(\arg)$;

where $p=\frac{N}{\sqrt{U^2+V^2}}$.

Also clearly φ, h, f satisfy all the conditions of **Hsu**'s theorem.

Now at some point $(\tau_0,\psi_0), h(\tau,\psi)$ to be stationary, if require $\left(\frac{\partial h}{\partial \tau}\right)_{(\tau_0,\psi_0)} = 0$ and $\left(\frac{\partial h}{\partial \psi}\right)_{(\tau_0,\psi_0)} = 0$

This again implies that, $r\cos(\psi_0-\varphi)-\frac{Z_1\tau_0}{\sqrt{\sec^2\psi_0-\tau_0^2}}=0$ and

$$\tau_0 r\sin(\psi_0-\varphi)=\frac{Z_1\sec^2\psi_0\tan\tau_0}{\sqrt{\sec^2\psi_0-\tau_0^2}}$$

Hence $\psi_0=\beta_0-\alpha=\tan^{-1}\left(-\frac{X_2Y_2}{\rho_1^2}\right)$ and $\tau_0=\frac{X_2Z_1\sqrt{\rho_1^4+(X_2Y_2)^2}}{\rho_1^3R_1}$

Where $\rho_1^2=Y_2^2+Z_1^2, R_1^2=X_1^2+\rho_1^2, X_2=r\cos\varphi$ and $Y_2=r\sin\varphi$

Now, $h(\tau_0,\psi_0)=\frac{Z_1R_1}{\rho_1}$ and $\left(\frac{\partial^2 h}{\partial\tau^2}\frac{\partial^2 h}{\partial\psi^2}-\left(\frac{\partial^2 h}{\partial\tau\partial\psi}\right)^2\right)_{(\tau_0,\psi_0)}=R_1^2\left[1+4\left(\frac{X_2Y_2Z_1R_1}{\rho_1^4+X_2^2Y_2^2}\right)\right]>0$

$$\therefore \varphi(\tau_0,\psi_0)=\frac{X_2Z_1\sqrt{\rho_1^4+X_2^2Y_2^2}}{\rho_1^3R_1}\left(h_1+h_2e^{-i\frac{\tau_0\cos\beta_0}{\sqrt{U^2+V^2}}Nd}\right)K_0(\arg)$$

By **Hsu's** theorem we get

$$I_1=\frac{\pi\Phi(\tau_0,\psi_0)[f(\tau_0,\psi_0)]^n}{\sqrt[n]{[\frac{\partial^2 h}{\partial\tau^2}\frac{\partial^2 h}{\partial\psi^2}-(\frac{\partial^2 h}{\partial\tau\partial\psi})^2]_{(\tau_0,\psi_0)}}}\qquad \text{here } n=1$$

$$I_1=\frac{i\pi A_1N^3X_2Z_1\left(h_1+h_2e^{-i\frac{\tau_0\cos\beta_0}{\sqrt{U^2+V^2}}Nd}\right)}{(U^2+V^2)}\exp\left(i\frac{Z_1R_1}{\rho_1}\right)$$

.

Similarly,

$$I_2=\frac{\pi A_1N^2\left(h_1+h_2e^{-i\frac{\tau_0\cos\beta_0}{\sqrt{U^2+V^2}}Nd}\right)}{(U^2+V^2)}\exp\left(i\frac{Z_1R_1}{\rho_1}\right)$$

Now, Real part of $I_1=\frac{\pi A_1N^3X_2Z_1}{(U^2+V^2)}\left\{h_2\sin\left(\frac{Z_1R_1}{\rho_1}-\frac{\tau_0\cos\beta_0}{\sqrt{U^2+V^2}}Nd\right)+h_1\sin\left(\frac{Z_1R_1}{\rho_1}\right)\right\}$

$$\text{Real part of } I_2 = \frac{\pi A_1 N^2}{\left(U^2+V^2\right)}\left\{h_2 \cos\left(\frac{Z_1R_1}{\rho_1} - \frac{\tau_0 \cos\beta_0}{\sqrt{U^2+V^2}} Nd\right) + h_1 \cos\left(\frac{Z_1R_1}{\rho_1}\right)\right\}$$

Substituting the values of I_1 and I_2 in the equations (26) and (27) respectively, we get

$$w'\left(X_2, Y_2, Z_1\right) = -e^{\frac{\left(g-R^{\bullet}\gamma\right)z}{2R^{\bullet}\bar{T}}} \frac{A_1 abN^3 X_2 Z_1}{\rho_1 R_1 (U^2+V^2)}\left[h_2 \sin\left(\frac{Z_1R_1}{\rho_1} - \frac{\tau_0 \cos\beta_0}{\sqrt{U^2+V^2}} Nd\right) + h_1 \sin\left(\frac{Z_1R_1}{\rho_1}\right)\right]$$

$$\eta'\left(X_2, Y_2, Z_1\right) = e^{\frac{\left(g-R^{\bullet}\gamma\right)z}{2R^{\bullet}\bar{T}}} \frac{A_1 abN^2}{(U^2+V^2)}\left[h_2 \cos\left(\frac{Z_1R_1}{\rho_1} - \frac{\tau_0 \cos\beta_0}{\sqrt{U^2+V^2}} Nd\right) + h_1 \cos\left(\frac{Z_1R_1}{\rho_1}\right)\right]$$

where, $$A_1 = \frac{X_2 Z_1 \sqrt{\rho_1^{\,4} + \left(X_2Y_2\right)^2}}{2\rho_1^{\,3} R_1^{\,2} \sqrt{1+4\left(\frac{X_2Y_2Z_1R_1}{\rho_1^{\,4} + X_2^{\,2}Y_2^{\,2}}\right)^2}} K_0\left(Arg_0\right)$$

$$Arg_0 = \frac{\left(NX_2Z_1\right)\sqrt{(a^2(U\rho_1^{\,2} - VX_2Y_2)^2 + b^2(V\rho_1^{\,2} + UX_2Y_2)^2)}}{\rho_1^{\,3} R_1\left(U^2+V^2\right)},$$

$$\rho_1^{\,2} = Y_2^{\,2} + Z_1^{\,2}, \; R_1^{\,2} = X_2^{\,2} + \rho_1^{\,2},$$

$$X_2 = \frac{N\left(Ux+Vy\right)}{\left(U^2+V^2\right)}, \; Y_2 = \frac{N\left(Uy-Vx\right)}{\left(U^2+V^2\right)} \text{ and } Z_1 = \frac{Nz}{\sqrt{U^2+V^2}}.$$

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