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Website: www.aarf.asia Email : editor@aarf.asia , editoraarf@gmail.com

AXIAL VIBRATION OF ELASTICALLY SUPPORTED BEAMS

Oktay Demirdag

Civil Engineering Department, Pamukkale University, Turkey

ABSTRACT

In this study free axial vibration of an elastically supported beam is analyzed. The supports are modeled by elastic translational springs. The frequency values for the first three vibration modes of the beam are obtained for various values of spring constants and presented in the tables. The frequency values for the spring constants of zero and infinity are also compared, respectively, with the ones of free and fixed beams and nearly the exact values are obtained with negligible error percentages.

Keywords: Axial vibration, Beam, Elastic Support, Frequency

1. Introduction

In practice, the representation of a beam by a discrete model is an idealized model; however, in fact, beams have continuously distributed mass and elasticity. Mostly, especially for the axially vibration, beams are modeled as continuous systems having infinite number of degreed of freedom [1-6].

In this study, the free vibration analysis of an elastically supported axially vibrating beam is made. The elastic springs against translation are used to model the supports. The differential equation of motion of the axially vibrating beam is solved by separation of variables method [7] and the displacement function is obtained. The boundary conditions are written for the elastic supports. The natural frequencies for the first three modes are obtained for the various values of the spring constants. The results obtained for the spring constant value of zero are compared with the frequency values of free beam whereas the ones for the spring constant value of infinity are compared with the frequency values of fixed beam. In addition, the frequencies obtained for the left end spring constant value of infinity and the right end spring constant value of zero are compared with the frequency values of cantilever beam. The axially vibrating beam considered in the study is assumed to be homogeneous and isotropic.

2. Solution of Equation of Motion for an Axıally Vıbratıng Beam

An axially vibrating beam, given in Figure.1, with the distributed mass m, the length L, the modulus of elasticity E, the cross-section area A and the axial rigidity AE has a dimensional differential equation of motion for free vibration as [8]

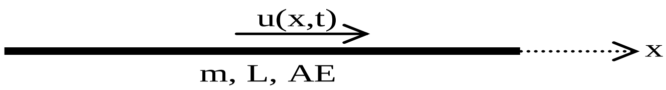

Figure 1: An Axially Vibrating Beam

$$\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{m}{AE}\frac{\partial^2 u(x,t)}{\partial t^2} = 0$$

(1)

where u(x,t) is the displacement function of the beam in terms of both displacement x and time t. Application of the separation of variables method to Eq. (1) as in the form of Eq. (2) is commonly used in vibration analysis of beams.

$$u(x,t) = X(x).T(t) = X(x).[A.\sin(\omega t) + B.\cos(\omega t)]$$

(2)

In Eq. (2), X(x) is the eigenfunction named as shape function, T(t) is time function, ω is the eigenvalue of the solution named as natural frequency and A, B are constants.

The derivatives used in Eq. (1) can, therefore, be written as

$$\frac{\partial^2 u(x,t)}{\partial x^2} = u''(x,t) = X''(x).[A.\sin(\omega t) + B.\cos(\omega t)] = X''(x).T(t)$$

(3)

$$\frac{\partial^2 u(x,t)}{\partial t^2} = \ddot{u}(x,t) = X(x).(-\omega^2)[A.\sin(\omega t) + B.\cos(\omega t)] = -\omega^2.X(x).T(t)$$

(4)

where ($^{//}$) and ($\ddot{\ }$) denote the second order derivative due to x and t, respectively. Substitution of Eq. (3) and Eq. (4) in Eq. (1) gives the governing equation of motion in the form as

$$X''(x).T(t) + \frac{m\omega^2}{\mathrm{AE}} X(x).T(t) = 0 \quad X''(x) + \frac{m\omega^2}{\mathrm{AE}} X(x) = 0 \quad 0 \le x \le L$$

(5)

for $\alpha^2 = \frac{m\omega^2}{\mathrm{AE}}$ $$X''(x) + \alpha^2 X(x) = 0$$

(6)

The characteristic equation and the solution of Eq. (6) is given as follows as D being d/dz:

$$D^2 + \alpha^2 = 0 \quad \rightarrow \quad D_{1,2} = \pm i\alpha \tag{7}$$

$$X(x) = C_1.\sin(\alpha x) + C_2.\cos(\alpha x) \tag{8}$$

Eq. (8) gives the shape function of the axially vibrating beam due to the displacement variable, x. Therefore, from Eq. (2), the displacement function of the axially vibrating beam has the form of Eq. (9).

$$u(x,t) = X(x).T(t) = [C_1.sin(\alpha x) + C_2.cos(\alpha x)].T(t)$$

(9)

3. Boundary Conditions

Figure 2: Elastically Supported Axially Vibrating Beam

Two boundary conditions have to be written for the elastically supported beam in Figure.2 since two integration constants (C_1, C_2) are obtained in the solution of second order differential equation of motion. The boundary conditions written for the left and the right ends of axially vibrating beam are given, respectively, as [9]

for x=0 $$N(x=0,t) = AE\frac{\partial u\,(x=0,t)}{\partial x} = AEu'(x=0,t) = k_0.u(x=0,t)$$

(10)

for x=L $$N(x=L,t) = AE\frac{\partial u\,(x=L,t)}{\partial x} = AEu'(x=L,t) = -k_L.u(x=L,t)$$

(11)

where k_o and k_L are the spring constant values of, respectively, the left end and the right end supports, N(x,t) is the axial force. If Eq. (9) and its derivative are substituted into Eq. (10) and Eq. (11) one gets the following relation between the coefficient matrix and the integration constants.

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} . \begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad \rightarrow \quad [k].\begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} = \{0\}$$

(12)

$$|k| = \begin{vmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{vmatrix} = 0$$

(13)

where $[k]$ is the coefficient matrix and $k_{11} = \propto$, $k_{12} = \frac{-k_0}{AE}$, $k_{21} = \propto . \cos(\propto L) + \frac{k_L}{AE} . \sin(\propto L)$, $k_{22} = -\propto . \sin(\propto L) + \frac{k_L}{AE} . \cos(\propto L)$. For non-trivial solution equating the determinant of the coefficient matrix of Eq. (12) to zero, as in Eq. (13), will give the eigenfrequencies of the axially vibrating beam with elastic supports. These frequencies are computed by a program written by the author considering the secant method [10].

4. Numerical Analysis

The first three natural frequencies of the axially vibrating beam with elastic supports are calculated for the k_0 and k_L values of 0, 10^3, 10^6, 10^9, 10^{12} and 10^{18}, the beam length of L=1 m. and the modulus of elasticity of E=2100000 kg/cm^2. IPB-100, IPB-300 and IPB-600 profiles are used for numerical analysis with the mechanical properties given in Table 1 where h is height, G is weight per length, A is cross-section area and AE is axial rigidity of the corresponding profile. The distributed mass of the beam m is calculated from G/g as g being the acceleration of gravity with the value of 981 cm/sn^2.

Table 1: The Mechanical Properties of the Profiles Used in This Study

Profile	h (cm)	G (kg/cm)	A (cm^2)	AE (kg)
IPB100	10	0.081	10.3	21630000
IPB300	30	0.422	53.8	112980000
IPB600	60	1.22	156	327600000

The frequency values computed due to different values of the spring constants for the both ends are presented in Table 2, Table 3 and Table 4 for, respectively, IPB-100, IPB-300 and IPB-600.

Table 2: Frequencies Computed due to Different Values of the Spring Constants for IPB-100

$k_0=k_L$	10^3	10^6	10^9	10^{12}	10^{15}	10^{18}
ω_1	492	11462	16073	16079.403691	16079.410640	16079.410647
ω_2	16095	24039	32145	32158.807382	32158.821280	32158.821294
ω_3	32167	37876	48218	48238.211074	48238.231921	48238.232041

Table 3: Frequencies Computed due to Different Values of the Spring Constants for IPB-300

$k_0=k_L$	10^3	10^6	10^9	10^{12}	10^{15}	10^{18}
ω_1	216	6355	16064	16100.077062	16100.113406	16100.113442
ω_2	16104	18558	32128	32200.154125	32200.226812	32200.226884
ω_3	32202	33577	48192	48300.231187	48300.340217	48300.340327

Table 4: Frequencies Computed due to Different Values of the Spring Constants for IPB-600

k_0=k_L	10^3	10^6	10^9	10^{12}	10^{15}	10^{18}
ω_1	127	3912	16020	16124.028685	16124.134224	16124.134330
ω_2	16126	17064	32039	32248.057370	32248.268448	32248.268659
ω_3	32249	32740	48058	48372.086055	48372.402672	48372.402989

In Table 5, the frequency values obtained for the spring constant values of zero and infinity (k_0=k_L=10^{18} in this study) are compared with the frequency values obtained for either free or fixed beams that have the same frequency equation.

Table 5: Comparison of Frequencies Computed for k_0=k_L=0 and k_0=k_L=∞ (10^{18} in this study) with the Frequencies of Free or Fixed Beam

		k_0=k_L=0 k_0=k_L=∞	$\omega_i = n_i \frac{\pi}{L}\sqrt{\frac{AE}{m}}$
IPB-100	ω_1	16079.410647	16079.410680
	ω_2	32158.821294	32158..821360
	ω_3	48238.231941	48238.232041
IPB-300	ω_1	16100.113442	16100.113434
	ω_2	32200.226884	32200.226868
	ω_3	48300.340327	48300.340302
IPB-600	ω_1	16124.134330	16124.134330
	ω_2	32248.268659	32248.268660
	ω_3	48372.402989	48372.402990

The frequency values computed due to different values of the spring constant k_0 and k_L=0 are presented in Table 6, Table 7 and Table 8 for, respectively, IPB-100, IPB-300 and IPB-600.

Table 6: Frequencies Computed due to Different Values of k_0 and k_L=0 for IPB-100

$k_0=k_L$	10^3	10^6	10^9	10^{12}	10^{15}	10^{18}
ω_1	348	6640	8038	8039.703585	8039.705322	8039.705324
ω_2	16087	20470	24114	24119.110754	24119.115966	24119.115971
ω_3	32163	35189	40190	40198.517923	40198.526609	40198.226618

Table 7: Frequencies Computed due to Different Values of k_0 and k_L=0 for IPB-300

$k_0=k_L$	10^3	10^6	10^9	10^{12}	10^{15}	10^{18}
ω_1	153	4214	8041	8050.047626	8050.056712	8050.056721
ω_2	16102	17407	24123	24150.142878	24150.170136	24150.170163
ω_3	32201	32903	40205	40250.238131	40250.283560	40250.283605

Table 8: Frequencies Computed due to Different Values of k_0 and k_L=0 for IPB-600

$k_0=k_L$	10^3	10^6	10^9	10^{12}	10^{15}	10^{18}
ω_1	90	2700	8036	8062.040754	8062.067138	8062.067165
ω_2	16125	16607	24108	24186.122261	24186.201415	24186.201495
ω_3	32249	32496	40179	40310.203768	40310.335692	40310.335824

In Table 9, the frequency values obtained for the spring constant values of $k_0=\infty$ and $k_L=0$ are compared with the frequency values obtained from the frequency equation of cantilever beam.

Table 9: Comparison of Frequencies Computed for $k_0=\infty$ and $k_L=0$ with the Frequencies of Cantilever Beam

		$k_0=\infty$ $k_L=0$	$\omega_i = \frac{(2n_i - 1)}{2} \frac{\pi}{L} \sqrt{\frac{AE}{m}}$
IPB-100	ω_1	8039.705324	8039.705340
	ω_2	24119.115971	24119.116020
	ω_3	40198.226618	40198.226700
IPB-300	ω_1	8050.056721	8050.056717
	ω_2	24150.170163	24150.170151
	ω_3	40250.283605	40250.283585
IPB-600	ω_1	8062.067165	8062.067165
	ω_2	24186.201495	24186.201495
	ω_3	40310.335824	40310.335825

5. Conclusions

In this study free longitudinal vibration of an elastically supported beam is made. The natural frequency values are obtained for different values of spring constants at both ends and presented in tables. It can be seen from Tables 2, 3, 4 and Tables 6, 7 and 8 that as the spring constant values increase through a value of 10^9 the frequency values rapidly increase, however, at the value of 10^9 the frequency values are so close to its ideal limit obtained from the frequency equation of ideal support condition, being free or fixed. As the spring constant value increases from 10^9 to theoretically infinity (practically 10^{18} in this study) the frequency values show gentle increase and at the spring constant value that represents infinity the frequency value reaches its limit value for the considering support. Increasing the height of the beam section causes a decrease in frequency values for the spring constant values of less than 10^9 and an increase for the spring constant values from 10^9 to infinity at which the frequency values show an increase no longer.

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